TEEGALA KRISHNA REDDY ENGINEERING COLLEGE

(UGC-Autonomous)

Approved by AICTE, Affiliated by JNTUH, Accredited by NAAC- 'A' Grade

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DEPARTMENT OF CIVIL ENGINEERING
LECTURE NOTES

ACADEMIC YEAR: 2023-24

REGULATION: R20

YEAR/SEMESTER: III - I

SUBJECT: **STRUCTURALANALYSIS– II**

SUBJECT CODE: **20CE5PC04**

FACULTY NAME: P.CHAITANYA

SYLLABUS:

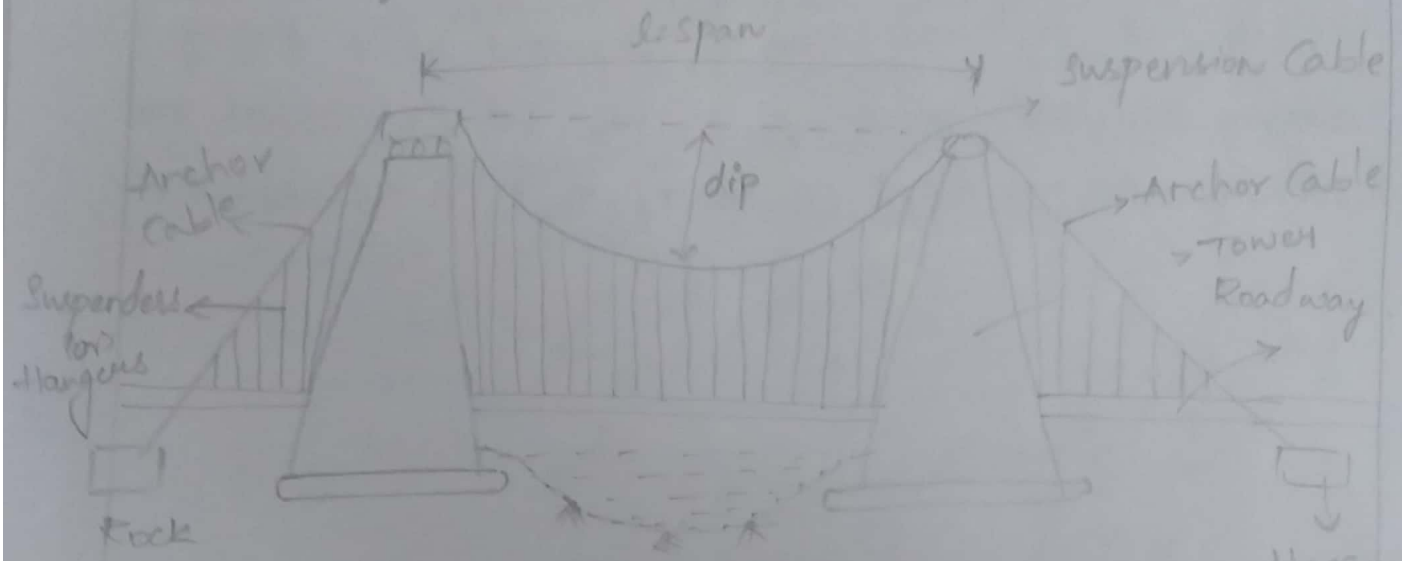
UNIT – II

Cables and suspension bridges: Equilibrium of a Suspension Cable subjected to concentrated loads and uniformly distributed loads –Length of a cable –Cable with different support levels –Suspension cable supports – Suspension Bridges –Analysis of Three Hinged Stiffening Girder Suspension Bridges.

UNIT-2

CABLES AND SUSPENSION BRIDGES

* Elevation of Suspension Bridge :



SUSPENSION BRIDGES :-

The Suspension Bridges are the Bridges that use ropes (or) Cables from the vertical suspenders to hold the weight of Bridge deck. The Bridge has cables suspended between the towers. The Suspension cables must be anchored at each end of Bridge.

The central sag (or) dip of cable varies from $\frac{1}{10}$ to $\frac{1}{15}$ th of Span. The Achor cables are provided to reduce the Bending moment in towers.

The cable is the main load bearing member which is flexible and curvature in nature. The cable consists of wire rope (or) parallel wires joined with links.

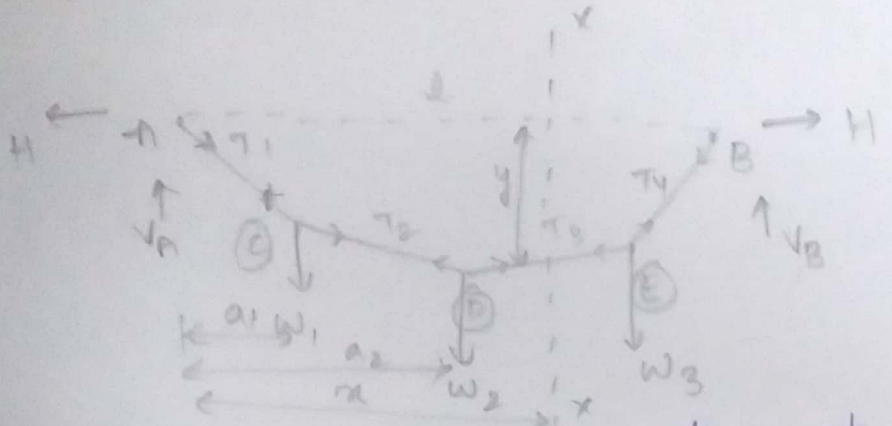
The cable can carry direct Tension force only, and the Bending moment at any point on the cable is zero.

The Suspension ~~Bridge~~ Bridge is constructed for the span more than 200m.

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EQUILIBRIUM OF A CABLE SUBJECTED TO POINT LOAD:

Consider a cable of span L supported at A & B & carrying the load system W_1, W_2, W_3. Let the Tensions are in parts AC, CD, DE & EB be T_1, T_2, T_3 & T_4 respectively.



Let V_A & V_B are vertical reactions at supports A & B & H is the Horizontal force developed at supports.

At section x-x, Let the deflection be y

The moment at section x-x

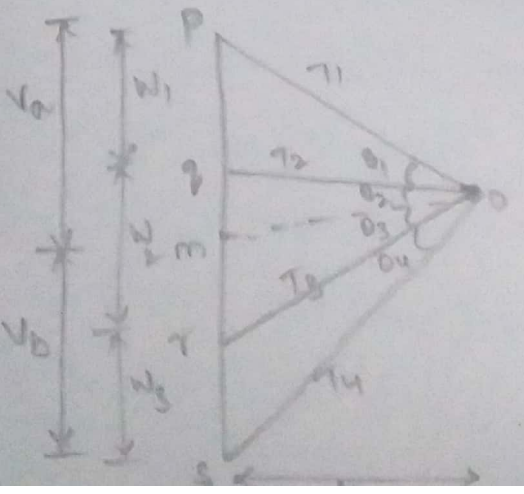
$$M_x = V_A \times x - W_1(x-a_1) - W_2(x-a_2) - H \times y = 0$$

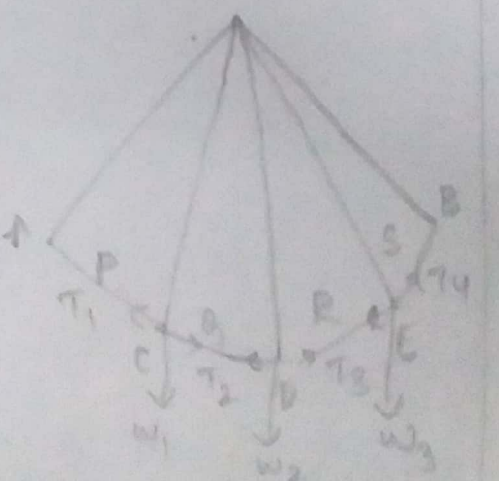
$$Hy = V_A \times x - W_1(x-a_1) - W_2(x-a_2)$$

where $H \times y$ = Beam Moment = M_{beam}

The loaded Cable can be analysed by using the above Equation

Shape of loaded Cord:

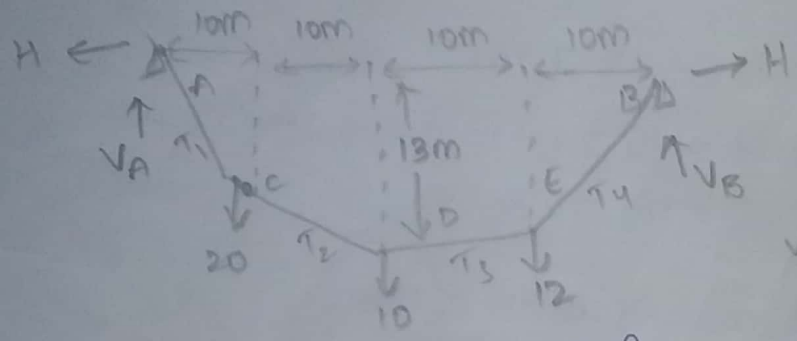


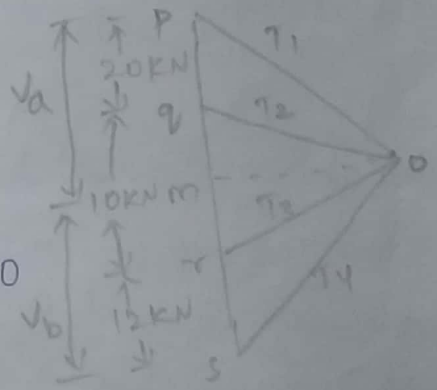


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18 A cord supported at its ends 40m apart carries loads of 20KN, 10KN, 12KN at distances 10m, 20m & 30m from left end. If the point on cord where 10KN load is supported 13m below the level of end supports. determine i) reactions at supports ii) Tensions in diff parts of cord. iii) Total length of cord.?

Sol.





Taking moments about support A,

$$V_B \times 40 = 20\times 10 + 10\times 20 + 12\times 30$$

$$\boxed{V_B = 19\ KN}$$

$$V_A + V_B = 20+10+12$$

$$V_A = (20+10+12) - 19 = 23\ KN$$

Taking the moment about D

$$23\times 20 = 20\times 10 + H\times 13$$

$$H = \frac{260}{13} = 20\ KN$$

The Tensions T_1 & T_2, T_3 & T_4 are $\boxed{T = \sqrt{V^2+H^2}}$

$T_1 = \sqrt{23^2+20^2} = 25.08\ KN$

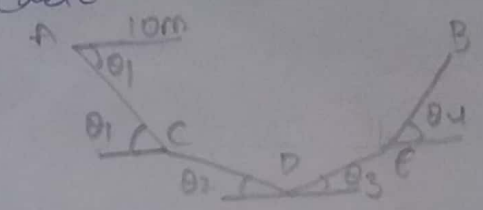
$T_2 = \sqrt{(23-20)^2+20^2} = 20.22\ KN$

$T_3 = \sqrt{(19-12)^2+20^2} = 21.19\ KN$

$T_4 = \sqrt{19^2+20^2} = 27.59\ KN$

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The length of different parts of cable



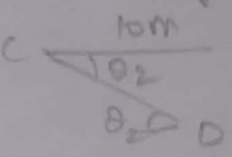
$\cos\theta_1 = \frac{10}{AC} \Rightarrow AC = 10 / \cos 49^\circ$

$= 10 \times \frac{1}{\cos 49^\circ}$

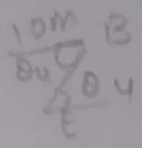
$= 15.24\text{m}$

Convert calculation into degrees

$\cos\theta_2 = \frac{10}{CD} \Rightarrow CD = \frac{10}{\cos 8^\circ 32'} = 10.11\text{m}$

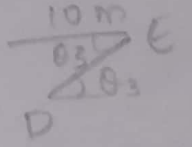


$DE = 10.60\text{m}$, $EB = 13.79\text{m}$

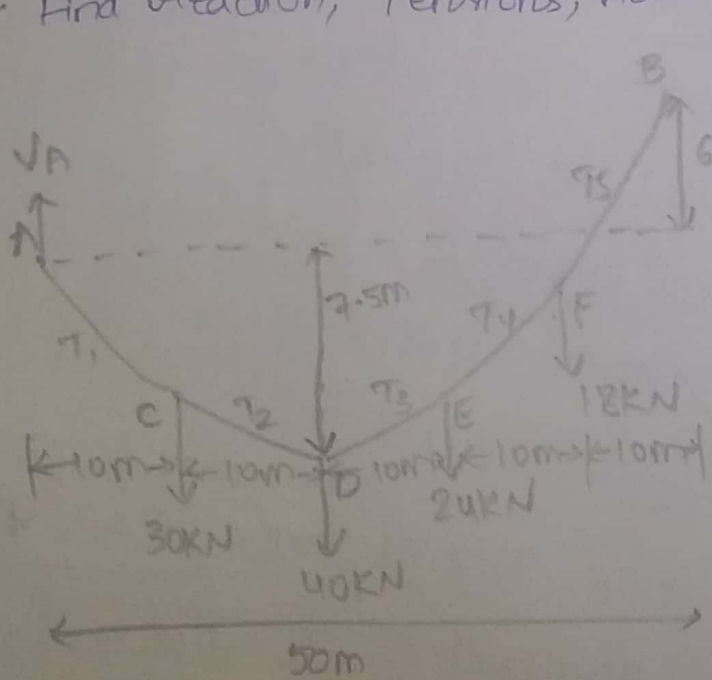


Total length of cord $= 15.24 + 10.11 + 10.60 + 13.79$

$= 49.74\text{ m}$.



(29) Fig shows a loaded cord ACDEFB of span 50m. The dip of cord at D is 7.5m below the left support A. The right support B = 6m higher than Support A. Find reaction, Tensions, inclination & total length of cable?



Sol Taking moment @ D from left

$V_A \times 20 = H \times 7.5 + 30 \times 10 =$ $\boxed{V_a = \frac{3}{8}H + 15}$

Taking moment @ D from right

$$V_b \times 30 = 13.5H + 24 \times 10 + 18 \times 20$$

$$30V_b = 13.5H + 600$$

$$\boxed{V_b = \frac{9}{20}H + 20}$$

Adding Eq ① & ②

$$\boxed{V_a + V_b = \frac{33}{40}H + 35}$$

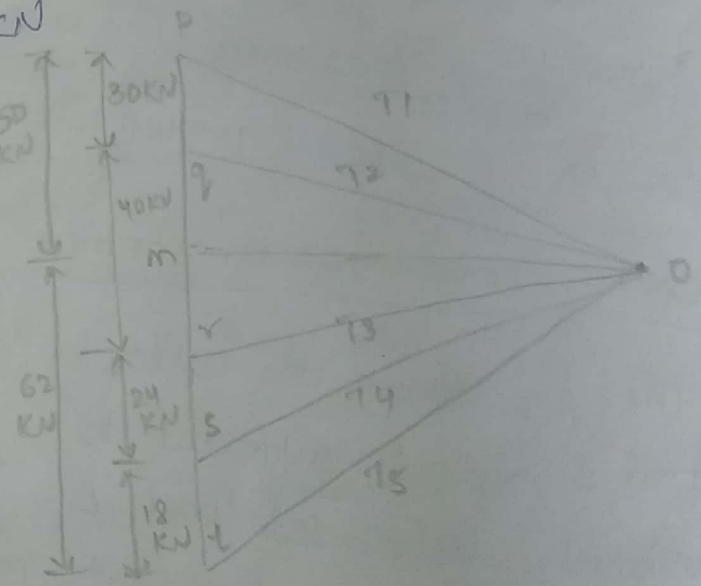
$$V_a + V_b = 30 + 40 + 24 + 18 = 112\,kN$$

$$\frac{33}{40}H + 35 = 112 \implies H = \frac{77 \times 40}{33} = \frac{280}{3}\,kN$$

$$\therefore \boxed{H = \frac{280}{3}\,kN}$$

$$V_a = \frac{3}{8} \times \frac{280}{3} + 15 = 50\,kN$$

$$V_d = \frac{9}{20} \times \frac{280}{3} + 20 = 62\,kN$$



Let T_1, T_2, T_3, T_4 & T_5 be the Tensions in parts AC, CD, DE, EF & FB

$\theta_1, \theta_2, \theta_3, \theta_4$ & θ_5 are the inclinations respectively.

$$T_1 = \sqrt{93.33^2 + 50^2} = 105.88\,kN$$

$$T_2 = \sqrt{93.33^2 + (50-30)^2} = 95.45\,kN$$

$$T_3 = \sqrt{93.33^2 + (62-(24+18))^2} = 95.45\,kN$$

$$T_4 = \sqrt{93.33^2 + 44^2} = 103.18\,kN$$

$$T_5 = \sqrt{93.33^2 + 62^2} = 112.05\,kN$$

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$\tan\theta_1 = \frac{50}{93.33} \Rightarrow \theta_1 = 28°11'$ $\tan\theta_3 = \frac{20}{93.33} \Rightarrow \theta_3 = 12°6'$

$\tan\theta_2 = \frac{20}{93.33} \Rightarrow \theta_2 = 12°6'$ $\tan\theta_4 = \frac{44}{93.33} \Rightarrow \theta_4 = 25°14'$

$\tan\theta_5 = \frac{62}{93.33} \Rightarrow \theta_5 = 33°36'$

$\therefore$ (*) $\boxed{\tan\theta = \frac{V_a}{H}}$

B.m at every point of cord is zero

B.m@C $= 50\times10 - \frac{280}{3}\times y_c = 0 \Rightarrow y_c = 5.36m$

B.m@E $= 62\times20 - 18\times10 - \frac{280}{3}(y_e + 6) = 0 \Rightarrow y_e = 5.36m$

B.m@F $= 62\times10 - \frac{280}{3}(y_f + 6) = 0 \Rightarrow y_f = 0.64m$

Length of cord

$AC = \frac{10}{\cos 28°11'} = 11.34m$ $DE = \frac{10}{\cos 12°6'} = 10.23m$

$CD = \frac{10}{\cos 12°6'} = 10.23m$ $EF = \frac{10}{\cos 25°14'} = 11.06m$

$FB = \frac{10}{\cos 33°36'} = 12.00m$

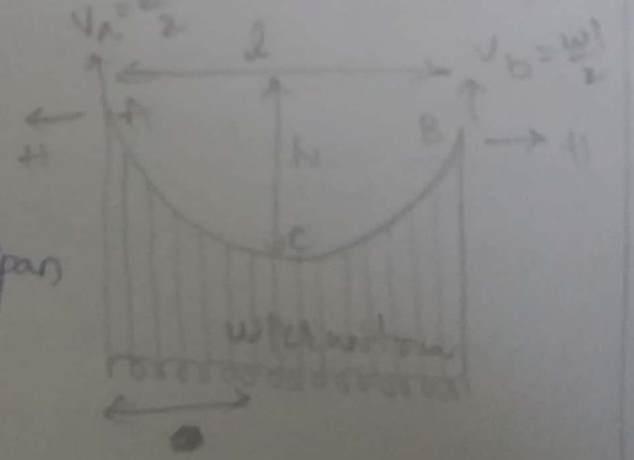
length of cable $= 11.34 + 10.23 + 10.23 + 11.06 + 12.00 = 54.86m$

(*) CABLE CARRYING UNIFORMLY DISTRIBUTED LOAD

The fig shows the cable of span l & dip h subjected to uniformly distributed load of w per unit run on horizontal span

V_a & V_b are vertical reactions

$V_a = V_b = \frac{wl}{2}$



Taking moments about C

$$-H \times h + w \times \frac{l}{2} \times \frac{l}{4} = \frac{wl}{2} \times \frac{l}{2}$$

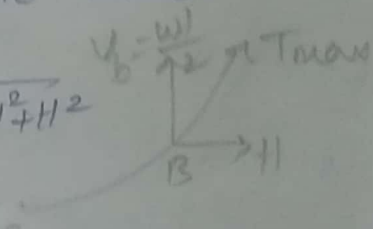
$$\therefore \boxed{H = \frac{wl^2}{8h}}$$

Maximum Tension T_{max} = Resultant reaction at A or B

$$= \sqrt{\left(\frac{wl}{2}\right)^2 + \left(\frac{wl^2}{8h}\right)^2} = \frac{wl}{2}\sqrt{1 + \frac{l^2}{16h^2}}$$

Let T be the Tension at any pt of cable.

Consider Equilibrium of part CB of cable. where θ be the inclination.



At lowest point c of cable, Tension $T = \sqrt{V^2 + H^2}$

At centre the middle of cable $V = 0$

$$T = \sqrt{0 + H^2} = T = H$$

@ point C $\Rightarrow \boxed{T_C = H}$

The Minimum Tension $T_C = H$ (@ middle of cable)

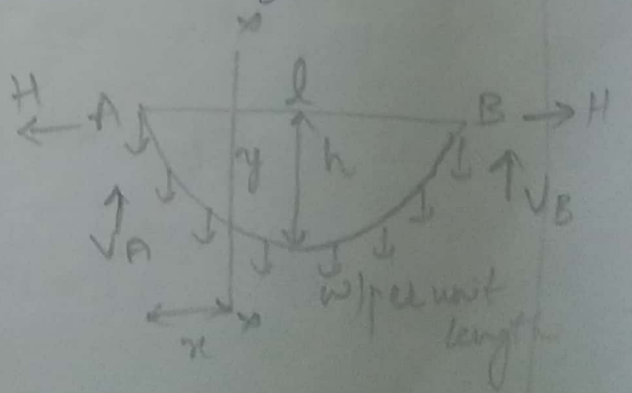
The Maximum Tension $T_{max} = \frac{wl}{2}\sqrt{1 + \frac{l^2}{16h^2}}$ (@ each Support)

The Shape of loaded Cord:

Consider a ~~par~~ cable of support A & B subjected to uniformly distributed load over the cable w/unit length of span l and dip h.

Let us consider a section x-x at distance of x and dip is y. as shown in fig.

$\therefore V_A$ & V_B are vertical reactions



$$V_A = V_B = \frac{wl}{2}$$

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B.M @ $x = 0$

$\Rightarrow V_A \times x - w \times x \times \frac{x}{2} - H \times y = 0$

$\Rightarrow \frac{wl}{2} \times x - \frac{wx^2}{2} = Hy$

$= \frac{wx}{2}(l-x) = \frac{wl^2}{8h} \times y$

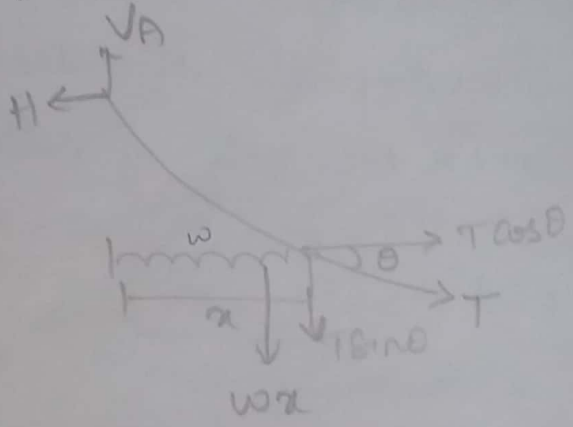
$\Rightarrow y = \frac{4h}{l^2} x(l-x)$

$\therefore$ the Equation of parabola $y = \frac{4h}{l^2} x(l-x)$

Another method ~~LENGTH OF THE CABLE~~ :

Consider the portion on left side of section x-x

Let θ be the slope. Then



$\Sigma H = 0 \quad T\cos\theta = H \rightarrow (1)$

$\Sigma V = 0 \quad T\sin\theta = V_A - wx$

(or) $T\sin\theta = \frac{wl}{2} - wx \rightarrow (2)$

From (1) & (2)

$\therefore \tan\theta = \left[\frac{wl}{2} - wx\right] \times \frac{1}{H}$

i.e., $\frac{dy}{dx} = \left[\frac{wl}{2} - wx\right] \times \frac{1}{H}$

$\therefore y = \left[\frac{wl}{2} x - \frac{wx^2}{2}\right] \times \frac{1}{H} \Rightarrow y = \frac{wx(l-x)}{2H}$

Sub $H = \frac{wl^2}{8h}$, we get

$y = \frac{wx(l-x)}{2 \times wl^2} \times 8h = \frac{4hx(l-x)}{l^2}$

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* Length of the Cable when Supported at Same level (5)

we Know that

$$y = \frac{4h}{l^2} x(l-x)$$

The shape of the cable is a parabola

To find length of the cable at any curve

$$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$= \sqrt{1+\frac{d^2y}{dx^2}} = 1+\left(\frac{d^2y}{dx^2}\right)^{1/2}$$

$$\Rightarrow 1+\frac{1}{2}\left(\frac{d^2y}{dx^2}\right) \Rightarrow 1+\frac{1}{2}\left[\frac{4h}{l^2}(l-2x)\right]^2$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{4h}{l^2}x(l-x)\right)$$

$$= \frac{d}{dx}\left[\frac{4h}{l^2}(lx-x^2)\right]$$

$$= \frac{d}{dx}\left[\frac{4h}{l^2}(lx-x^2)\right] = \frac{4h}{l^2}(l-2x)$$

(As per Binomial theorem Equation, $(1+x)^n = 1+nx+\frac{n(n-1)}{2}x^2+\cdots$ neglecting higher powers gives $(1+nx)$)

∴ length of cable is given by $ds = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2} dx$

$ds = \left[1+\frac{1}{2}\times\frac{d^2y}{dx^2}\right]dx$

$$L = \int_0^l ds = \int_0^l \left[1+\frac{1}{2}\frac{16h^2}{l^4}(l^2+4x^2-4lx)\right]dx$$

$$= \left[x+\frac{8h^2}{l^4}\left(l^2x-\frac{4lx^2}{2}+\frac{4x^3}{3}\right)\right]_0^l$$

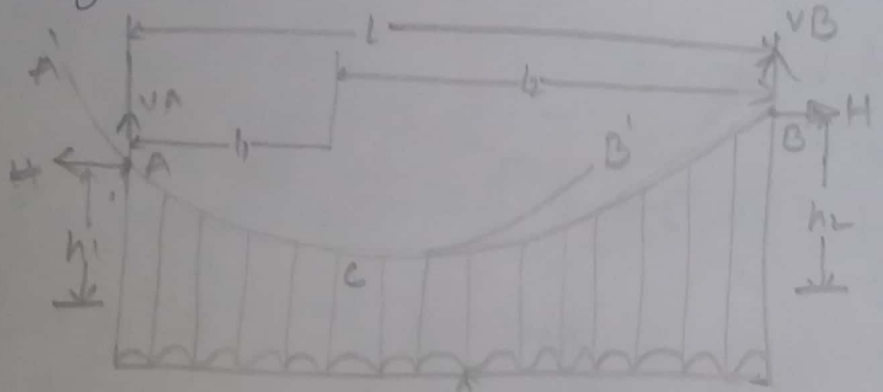
$$= l+\frac{8h^2}{l^4}\times l^3\left[1+2+\frac{4}{3}\right]$$

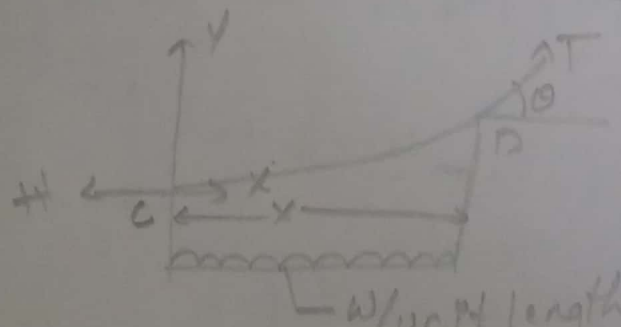
$$\therefore \boxed{L = l+\frac{8h^2}{3l}}$$

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* length of the Cable, when supported at Different level.

We know that
Cable which is supported at A and B. A is h_1 meters above the lowest point C and B is h_2 meters above C. This is subjected to a uniformly distributed load w per unit horizontal length over entire span l. Let the horizontal distance between AC be l_1 and between CB be l_2. Let the horizontal reaction at support be H.





A Cable can take only axial force, it cannot take bending moment and shear force. Hence at lowest point C, the axial force = H and there is no force in vertical direction.

Let D be a point where slope is θ. Taking lowest point C!

as origin and considering the equilbrium of portion CD, we get

$$T\cos\theta = H$$

$$T\sin\theta = wx$$

$$\tan\theta = \frac{wx}{H}$$

i.e., $$\frac{dy}{dx} = \frac{wx}{H}$$

$$y = w\frac{x^2}{2H} + c_1 \text{ ; where } c_1 \text{ is constant integration}$$

At C, $x = y = 0$

$$0 = 0 + c_1 \text{ or } c_1 = 0$$

$$y = w\frac{x^2}{2H}.$$

This is equation of Parabola, Hence the Cable is having a Parabolic shape. Apply $y = w\frac{x^2}{2H}$ to point A & B. we get

$$h_1 = \frac{wl_1^2}{2H} \text{ and } h_2 = \frac{wl_2^2}{2H}$$

$$\frac{h_1}{h_2} = \frac{l_1^2}{l_2^2} \quad (\text{or}) \quad \frac{\sqrt{h_1}}{\sqrt{h_2}} = \frac{l_1}{l_2}$$

$$\therefore \frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} = \frac{l_1}{l_1+l_2} = \frac{l_1}{l}$$

$$\therefore l_1 = l\frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}$$

$$\therefore l_2 = l\frac{\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}.$$

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To find H, Calculate moment at C, If it is calculated from left-hand side.

$$V_A l_1 - H h_1 - \frac{w l_1^2}{2} = 0$$

(or) $$V_A = \frac{w l_1}{2} + \frac{h_1}{l_1} = H \quad \Big\} \rightarrow (1)$$

If Calculated right-hand side

$$V_B l_2 - H h_2 - \frac{w l_2^2}{2} = 0$$

(or) $$V_B = \frac{w l_2}{2} + \frac{h_2}{l_2} = H \quad \Big\} \rightarrow (2)$$

Add Eq (1) & (2) we get

$$V_A + V_B = \frac{w}{2}(l_1 + l_2) + \left[\frac{h_1}{l_1} + \frac{h_2}{l_2}\right] H$$

$$\cdot \frac{w}{2} l \left[\frac{h_1}{l_1} + \frac{h_2}{l_2}\right] H$$

$V_A + V_B = wl$, Total downward load

$$wl = \frac{w}{2} l + \left[\frac{h_1}{l_1} + \frac{h_2}{l_2}\right] H$$

(or) $$\frac{wl}{2} = \left[\frac{h_1}{l_1} + \frac{h_2}{l_2}\right] H.$$

Substitute l_1 & l_2 values.

$$H = \frac{wl}{2\left[\frac{h_1(\sqrt{h_1} + \sqrt{h_2})}{l\sqrt{h_1}} + \frac{h_2(\sqrt{h_1} + \sqrt{h_2})}{l\sqrt{h_2}}\right]}$$

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$$H = \frac{w l^2}{2\left[\sqrt{h_1}\left(\sqrt{h_1}+\sqrt{h_2}\right) + \sqrt{h_2}\left(\sqrt{h_1}+\sqrt{h_2}\right)\right]}$$

$$H = \frac{w l^2}{2\left(\sqrt{h_1}+\sqrt{h_2}\right)^2}$$

from Eq (1) & (2) V_A and V_B can be found

length of cable ACB = $\frac{1}{2}$ × Sum of length of ACB' and A'CB

$$= \frac{1}{2} \times \left[2l_1 + \frac{8}{3}\frac{h_1^2}{2l_1} + 2l_2 + \frac{8}{3}\frac{h_2^2}{2l_2}\right]$$

$$= l_1 + l_2 + \frac{2}{3}\frac{h_1^2}{l_1} + \frac{2}{3}\frac{h_2^2}{l_2}$$

$$L = l + \frac{2}{3}\frac{h_1^2}{l_1} + \frac{2}{3}\frac{h_2^2}{l_2}.$$ //

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A cable carrying a load of 10KN per meter run of horizontal span, is stretched between supports 100m apart. The supports are at same level and the dip = 8m. Find greatest & least tensions in cable.

Vertical reaction $V = \frac{wl}{2} = \frac{10 \times 100}{2} = 500\,KN$

Horizontal reaction $H = \frac{wl^2}{8h} = \frac{10 \times 100^2}{8 \times 8} = 1562.5\,KN$

Max Tension $T_{max} = \sqrt{V^2 + H^2} = \sqrt{500^2 + 1562.5^2}$

$= 1640.55\,KN$

Min Tension $T_{min} = H$

$= 1562.5\,KN$

Q) A Cable of span L and dip h is subjected to a uniformly distributed load w per run of horizontal span. If the dip be considered as small compared with the span show that the difference between the greatest and the least tension is approximately wh.

Sol Each Vertical reaction $= V = \frac{wL}{2}$;

Horizontal reaction $= H = \frac{wL^2}{8h}$

Maximum Tension $= T_{max} = \sqrt{V^2+H^2}$

$$= \sqrt{\left(\frac{wL}{2}\right)^2 + \left(\frac{wL^2}{8h}\right)^2} \Rightarrow \sqrt{\frac{w^2L^2}{4} + \frac{w^2L^4}{64h^2}}$$

$$\Rightarrow \sqrt{\frac{w^2L^4}{64h^2}\left[1+\frac{16h^2}{L^2}\right]}$$

$$= \frac{wL^2}{8h}\left[1+\frac{16h^2}{L^2}\right]^{1/2}$$

→ Binomial Eq $(1+n)^{1/2} = (1+\frac{1}{2}n)$

$$= \frac{wL^2}{8h}\left[1+\frac{1}{2}\cdot\frac{16h^2}{L^2}\right]$$

$$= \frac{wL^2}{8h}\left[1+\frac{8h^2}{L^2}\right] = \frac{wL^2}{8h} + wh$$

Minimum Tension $T_{min} = H = \frac{wL^2}{8h}$.

$$T_{max} - T_{min} = \frac{wL^2}{8h} + wh - \frac{wL^2}{8h}$$

$$= wh.$$

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Q) Find the Maximum possible span for a Cable supported at the ends at the same level allowing a central dip of $\frac{1}{10}$ the span and a permissible stress of 150 N/mm². Steel weighs 78000 N per cum. Assume that the Cable takes a parabolic profile.

Sol: let Maximum horizontal span be 'l' metres.

Dip of Cable = $h = \frac{l}{10}$ metres

length of Cable = $L = l + \frac{8}{3} \cdot \frac{h^2}{l}$

$= l\left[1 + \frac{8}{3} \cdot \left(\frac{h}{l}\right)^2\right]$

$L = l\left[1 + \frac{8}{3} \cdot \frac{1}{100}\right] = \frac{308}{300} l$

let Area of Cable A mm²

Weight of Cable = $wl = \frac{308}{300} l \cdot \frac{A}{1000^2} \times 78000$ N

$wl = 0.08008\ A\ L$ Newtons

Each Vertical reaction $V = \frac{wl}{2} = \frac{W}{2}$

Horizontal reaction $= H = \frac{wL^2}{8h} = \frac{W}{8}(10) = \frac{5}{4} wl$ → $\frac{WL}{8h}$

$\therefore$ Max. tension = $T_{max} = \sqrt{V^2 + H^2}$

$= \sqrt{\left(\frac{W}{2}\right)^2 + \left(\frac{5W}{4}\right)^2}$

$= 1.35\ wl$

$= 1.35 \times 0.08008\ A\ L$ Newtons

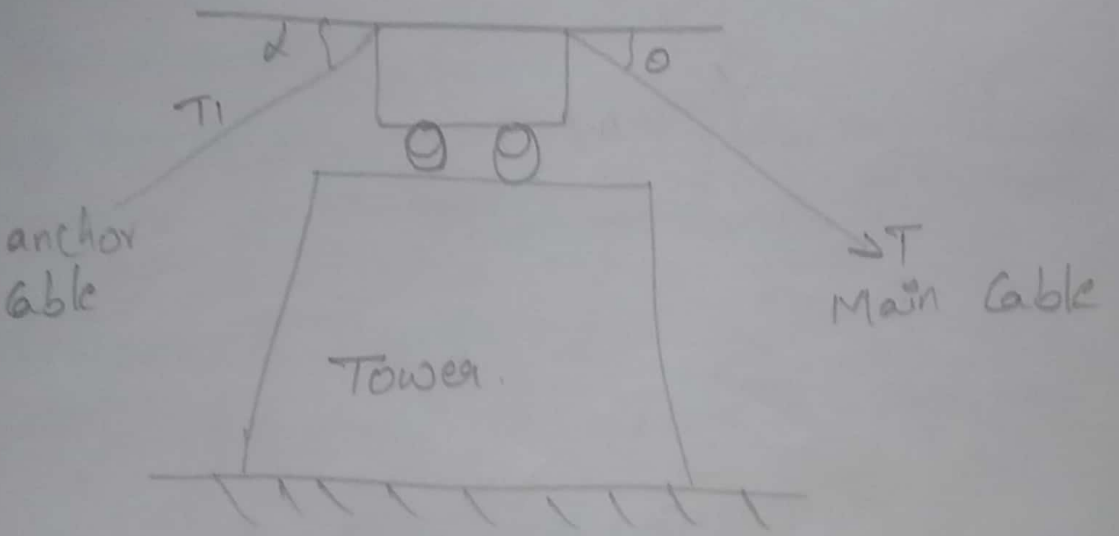
Maximum stress = $\frac{T_{max}}{A} = f_{max}$

$\therefore 1.35 \times 0.08008\ L = 150$

$L = 1387.5$ m

Roller Support

In this case, the suspension cable and the anchor cables are connected to a saddle resting on a tower. In this arrangement, the two cables need not have the same tension. Let T be the tension in main cable and T_1 in the anchor cable.



Assuming saddle to have frictionless rollers,

$$T_1 \cos\alpha = T\cos\theta = H$$

$$T_1 = T\frac{\cos\theta}{\cos\alpha}$$

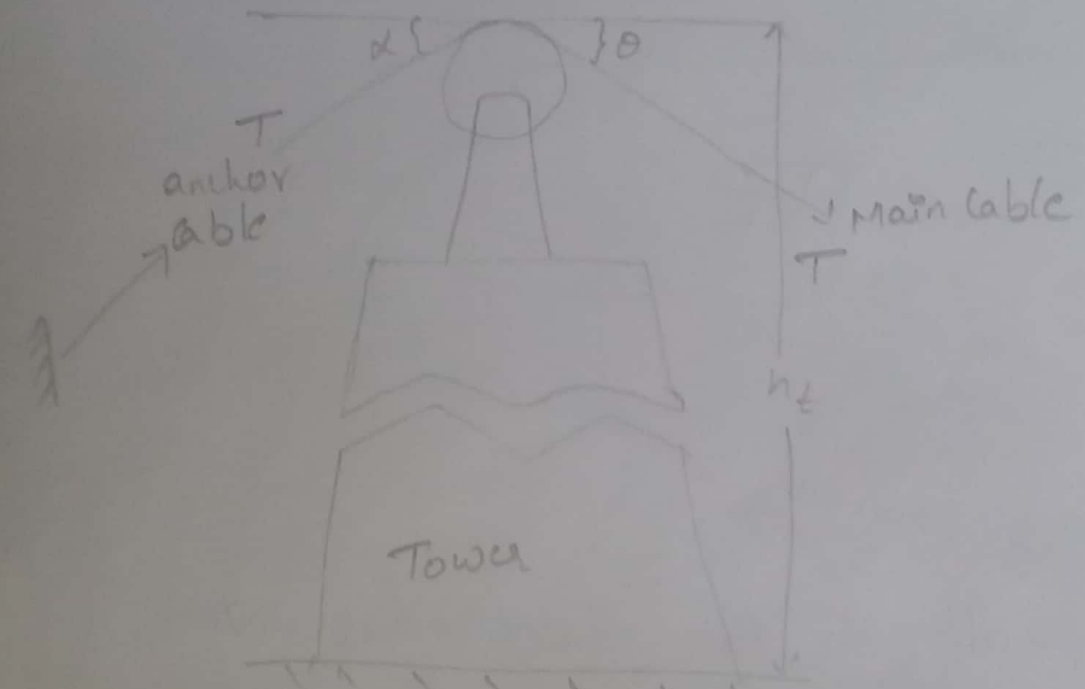
Since saddle is having frictionless rollers, there is no horizontal force and hence no bending moment on tower.

Vertical force on Tower $= T_1 \sin\alpha + T\sin\theta$

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Guided Pulley Support :-

let the inclination of main cable to horizontal be θ and anchor cable be α. Assuming Pulley as frictionless, tension in anchor cable is taken as same as tension in the main cable. let this tension be T.



Vertical load transmitted to tower

$$= T\sin\theta + T\sin\alpha$$
$$= T(\sin\theta + \sin\alpha)$$

Horizontal load transmitted to tower

$$= T\cos\theta - T\cos\alpha$$
$$= T(\cos\theta - \cos\alpha)$$

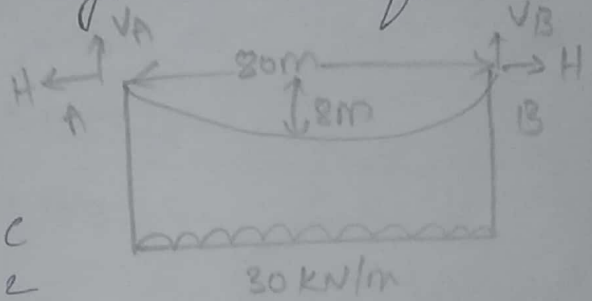
Bending moment on Tower

= Horizontal force on tower × Height of tower

$$= T(\cos\theta - \cos\alpha)h_1$$

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9) A Bridge cable is suspended from Towers 80m apart & carries a load of 30 KN/m on entire span. If max sag is 8m, calculate max Tension in cable. If cable is supported by saddles with are stayed by wires inclined at 30° to horizontal, determine the forces acting on towers. If same inclination passes over pulley, determine forces on towers? of back stay



∴ WKT

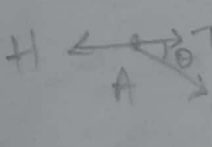
$$V_A = \frac{wl}{2} = \frac{30 \times 80}{2} = 1200\,KN$$

Taking moment @ central pt C

$$H = \frac{wl^2}{8h} = \frac{30 \times 80^2}{8 \times 8} = 3000\,KN$$

Max Tension @ supports

$$T_{max} = \sqrt{V^2 + H^2} = \sqrt{1200^2 + 3000^2} = 3231.1\,KN$$



$$T_{max}\cos\theta = H$$

$$\theta = \cos^{-1}\left(\frac{H}{T_{max}}\right) = \cos^{-1}\left(\frac{3000}{3231.1}\right)$$

$$= 21.80°$$

i) If cable is supported by saddle, the anchor cable tension T_1 is given by

$$T_1 \cos\alpha = T_{max}\cos\theta$$

$$T_1 \cos 30° = 3231.80 \times \cos 21.80°$$

$$T_1 = 3464.1\,KN$$



No Horizontal force on Tower,

Vertical force on Tower is

$$= T_1 \sin\alpha + T_{max}\sin\theta$$

$$= 3464.1 \sin 30° + 3231.1 \sin 21.80°$$

$$= 2931.98\,KN$$

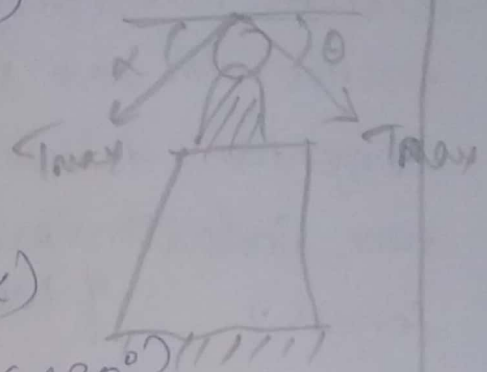
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ii) If cable is on pulley, the vertical force on Tower

$= T_{max}(\sin\alpha + \sin\theta)$

$= 3231.1(\sin 30 + \sin 21.80^\circ)$

$= 2815.48\ KN$



Horizontal force on Tower

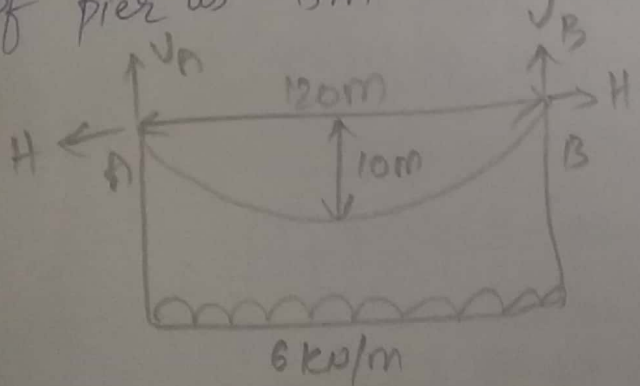
$= T_{max}(\cos\theta - \cos\alpha)$

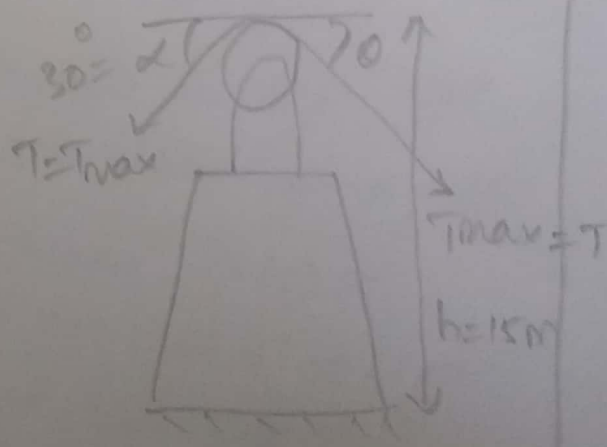
$= 3231.1(\cos 21.8^\circ - \cos 30^\circ)$

$= 201.82\ KN$

Assignment :-

A cable of span 120m & dip 10m carries a load of 6KN/m of horizontal span. Find the max Tension in cable & inclination of cable at support. Find forces transmitted to supporting pier. If cable passes over a pulley on top of pier. The anchor cable is 30° to horizontal. Determine B.M maximum if height of pier is 15m

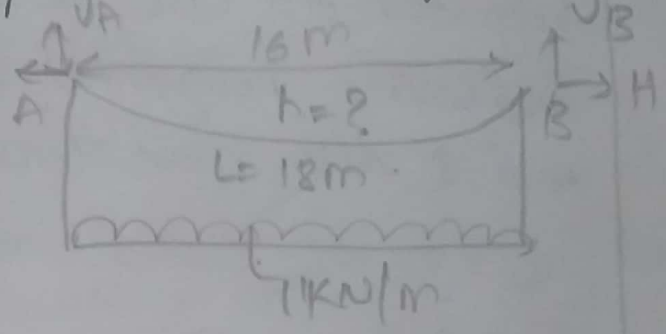




8) A light flexible cable 18m long is supported at two ends at same level. The supports are 16m apart. The cable is subjected to uniformly distributed load of 1KN/m of horizontal length over its entire span. Determine the reactions developed at the support?

Sol. The length of the cable

$$L = l + \frac{8}{3} \times \frac{h^2}{l}$$



$$18 = 16 + \frac{8}{3} \times \frac{h^2}{16}$$

$$\therefore h = 3.464\,m$$

$$V_A = \frac{wl}{2} = \frac{1 \times 16}{2} = 8\,KN \qquad H = \frac{wl^2}{8h}$$

$$H \times 3.464 = \frac{wl^2}{8} = \frac{1 \times 16^2}{8}$$

$$\therefore H = 9.237\,KN$$

$$T_{max} = \sqrt{V^2 + H^2} = \sqrt{8^2 + (9.237)^2} = 12.220\,KN$$

Its inclination to horizontal is given by

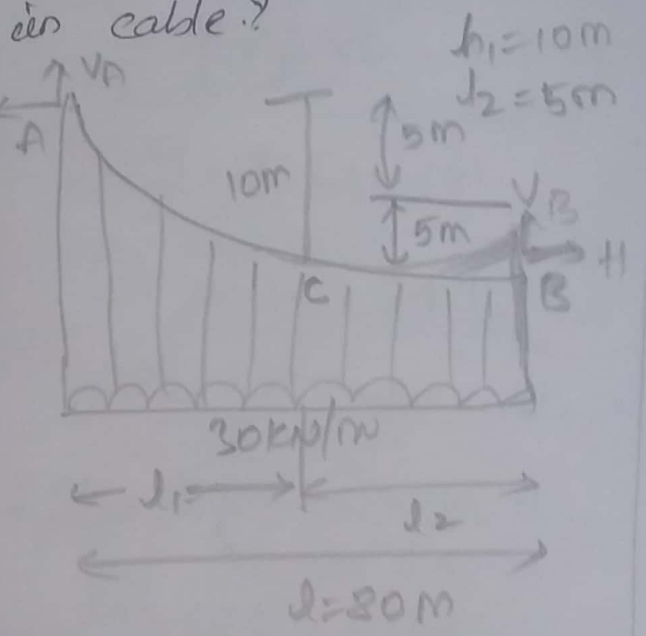
$$T_{max} \cos\theta = H$$

$$\theta = \cos^{-1}\left(\frac{H}{T_{max}}\right)$$

$$= \cos^{-1}\left(\frac{9.237}{12.220}\right) = 40.898^\circ$$

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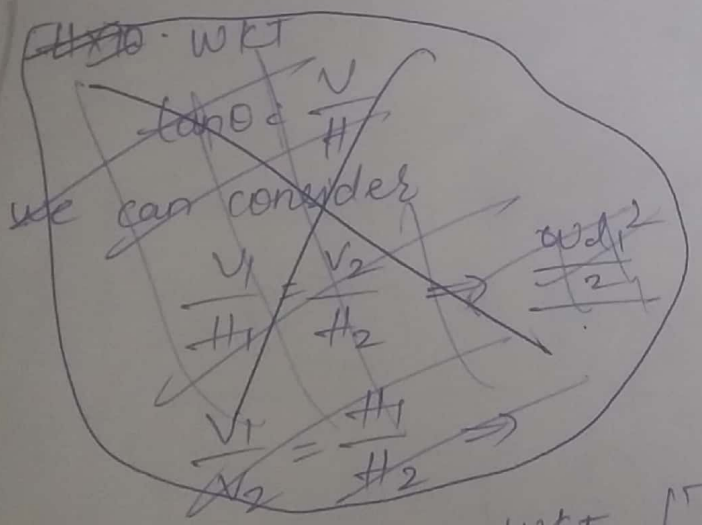
6) A cable is suspended from points A & B which are 80m apart horizontally and are at different levels. The point A being 5m vertically higher than the pt B. & lowest pt in cable is 10m below A. The cable is subjected to UDL of 30kN/m over horizontal span. Determine horizontal & vertical reactions @ each end & also max Tension in cable?



Sol V_A & V_B are vertical reactions @ A & B

H is Horizontal reaction

(The Equilibrium condition of moment about A & B is)



$y = \frac{wx^2}{2H}$

$h_1 = \frac{wl_1^2}{2H}$

From the Equation, WKT $\frac{h_1}{h_2} = \frac{l_1^2}{l_2^2}$

$h_2 = \frac{wl_2^2}{2H}$

$$\frac{10}{5} = \frac{l_1^2}{l_2^2} \Rightarrow l_1^2 = 2l_2^2$$

$$\Rightarrow l_1 = \sqrt{2}\, l_2$$

$$l_1 + l_2 = 80m$$

$$\Rightarrow l_2(\sqrt{2}+1) = 80m \Rightarrow l_2 = 33.137m$$

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$l_1 = 80 - 33.137 = 46.863\ m$ $\Rightarrow V_A \times l_1 - H \times y = 0$

$H = \frac{w l_1^2}{2h_1} = \frac{w \times 46.863^2}{20} = \frac{30 \times 46.863^2}{20}$ $\frac{w l_1^2}{2} = 10 \times H$

$= 3294.2\ kN$ $H = \frac{w l_1^2}{20}$

Vertical forces on AC,

$V_A = w \times l_1 = 30 \times 46.863 = 1405.89\ kN$

Vertical forces on CB,

$V_B = w l_2 = 30 \times 33.137 = 994.11\ kN$

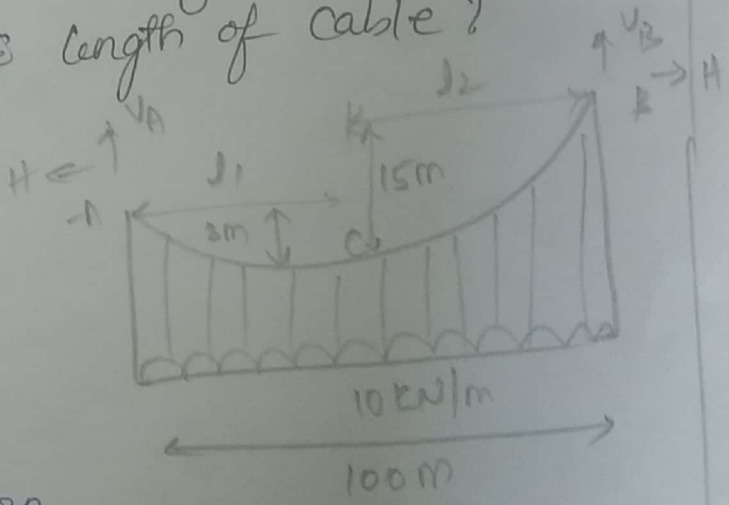
$\therefore V_A > V_B$, The max Tension occurs at support A

$\rightarrow T_{max} = \sqrt{V_A^2 + H^2} = \sqrt{1405.89^2 + 3294.2^2}$

$= 3581.66\ kN$

(8) A Cable of span 100m has its ends at heights 8m & 15m above the lowest pt of cable. It carries UDL of 10kN/m. Determine horizontal & vertical reactions at supports & length of cable?

Sol $h_1 = 8m,\ h_2 = 15m,\ l = 100m$



$\frac{h_1}{h_2} = \frac{l_1^2}{l_2^2}$

$\frac{8}{15} = \frac{l_1^2}{l_2^2} \Rightarrow \frac{l_1}{l_2} = \frac{\sqrt{8}}{\sqrt{15}}$

$= 0.730$

$\frac{l_1}{l_1 + l_2} = \frac{0.730}{1.730}$

$l_1 = \frac{0.73}{1.73} \times l \Rightarrow l_1 = \frac{0.73}{1.73} \times 100 \Rightarrow l_1 = 42.214m$

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$l_2 = 100 - 42.214 = 57.786m$

$V_A = wl_1 = 10 \times 42.214 = 422.14 KN$ $V_A \times l_1 = H \times 8$

$\therefore H = \frac{wl_1^2}{8h} = \frac{10 l_1^2}{8 \times 2} = \frac{10 \times 42.214^2}{8 \times 2}$ $\frac{wl_1^2}{2 \times 8} = H$

$= 1113.76 KN$ $\therefore H = \frac{wl_1^2}{8 \times 2}$

$V_B = wl_2 = 10 \times 57.786 = 577.86 KN.$

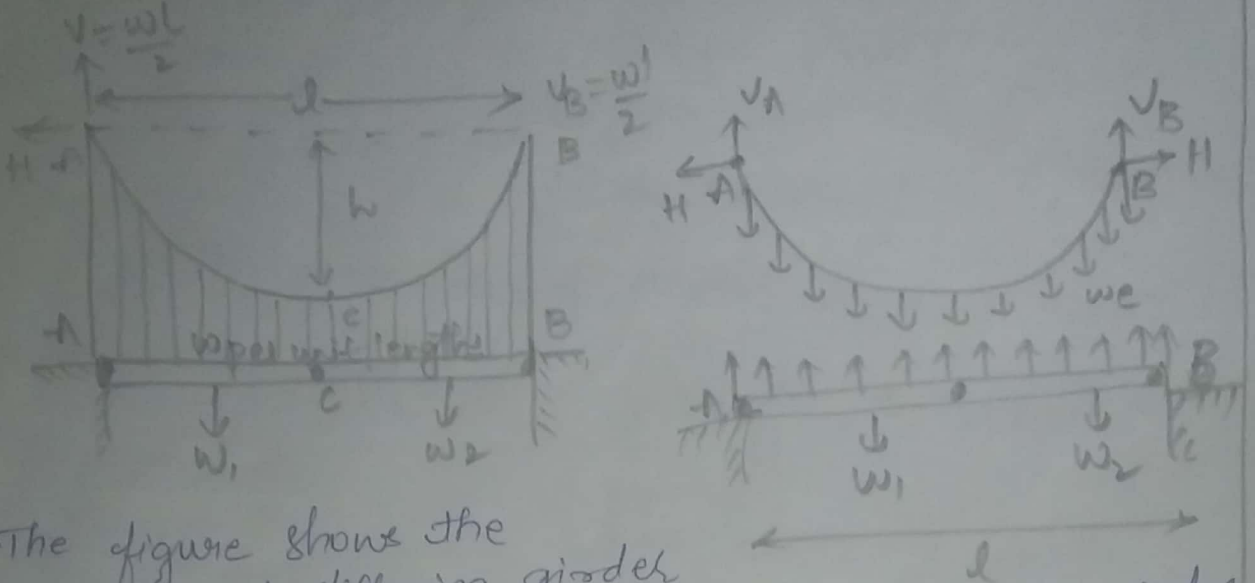
Length of cable $L = l + \frac{2}{3}\frac{h_1^2}{l_1} + \frac{2}{3}\frac{h_2^2}{l_1}$

$= 100 + \frac{2}{3} \times \frac{8^2}{42.214} + \frac{2}{3} \times \frac{15^2}{57.786}$

$= 103.607m$

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SUSPENSION BRIDGE WITH THREE-HINGED STIFFENING GIRDER



The figure shows the three-hinged stiffening girder supported at ends & connected to cable. Let the girder be subjected to any external load system w_1, w_2. Let w_e is the UDL transferred to cable.

Let us consider the Equilibrium of cable & stiffening girder seperately

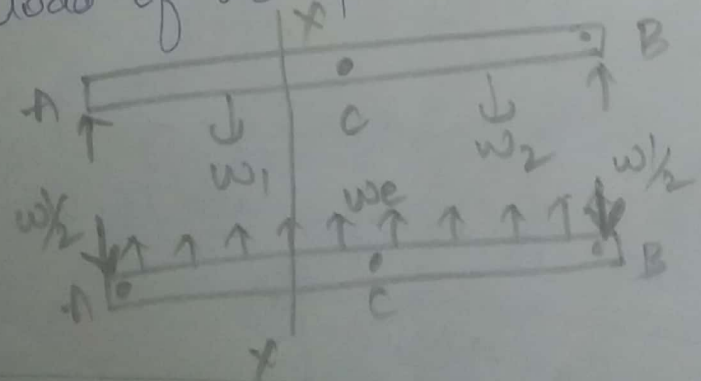
① Consider cable subjected to udl

Eq of cable $y = \frac{4h}{l^2} x(l-x)$

$V = \frac{w_e l}{2}$ @ each supports, $H = \frac{w_e l^2}{8h}$ @ each support

Max Tension $T_{max} = \sqrt{V^2 + H^2}$

② Consider Girder subjected to two load system w_1 & w_2 & udl load of w_e upward

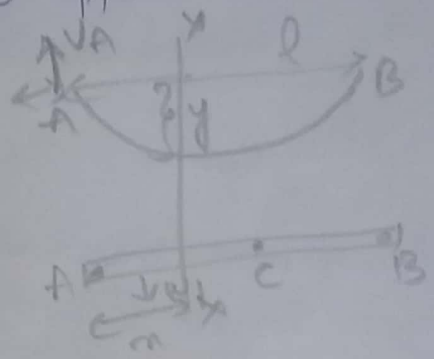


Consider any section x-x. at dist x from A.

$$M = M_x - \left[\frac{w_e l}{2} x - \frac{w_e x^2}{2}\right]$$

$$M = M_x - \frac{w_e}{2} x(l-x) \rightarrow (1)$$

M_x = B.M at the section of load applied at x.



$$\therefore Hxy = \frac{w_e l^2}{8h} \times \frac{4h}{l^2} x(l-x)$$

$$\therefore Hxy = \frac{w_e}{2} x(l-x) \rightarrow (2)$$

From (1) & (2)

we get $\boxed{M = M_x - Hy}$ (*)

∴ The Actual B.M at any section ⇒ (M)

∴ M = Beam moment − H moment

Considering the condition @ hinge C BM = 0

$$M = M_x - Hy = 0 \Rightarrow M_c - Hh = 0 \Rightarrow M_c = Hh$$

$$\therefore \boxed{H = \frac{M_c}{h}}$$

$$\therefore H = \frac{w_e l^2}{8h} \Rightarrow \boxed{w_e = \frac{8Hh}{l^2}}$$

∴ The shear force at any section of girder

$$S = S_x - \frac{w_e l}{2} - w_e x$$

$$\boxed{S = S_x - \frac{w_e}{2}(l-2x)} \rightarrow (1)$$

∴ S_x is the shear Force at the section of load applied.

slope at any section $= \tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$

$$H \cdot \tan\theta = \frac{w_e l^2}{8xh} \times \frac{4h}{l^2}(l-2x) = \frac{w_e}{2}(l-2x) \rightarrow (2)$$

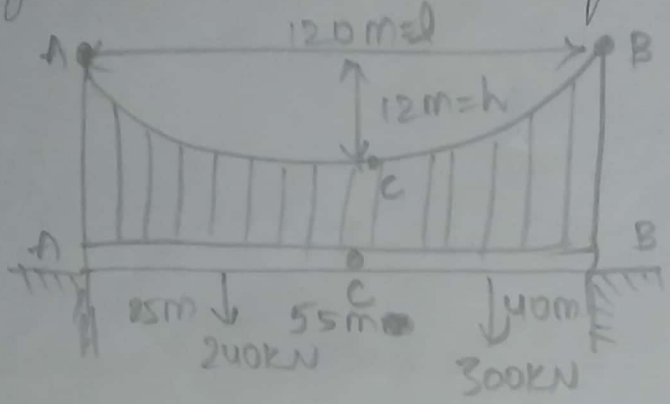
from (1) & (2)

we get $\boxed{S = S_x - H\tan\theta}$ (*)

The Actual S.F(S) at any section = Beam shear − $H\tan\theta$.

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9) The three-hinged stiffening girder of suspension bridge of span 120m is subjected to 2 point loads of 240KN & 300KN at dist 25m & 80m. from left end. Find SF & B.M for girder at dist 40m from left end. The cable has dip of 12m. Find max Tension of cable & Draw B.M diagram for girder.



Taking moment about A.

$V_b \times 120 = 240 \times 25 + 300 \times 80$

$\boxed{V_b = 250\,KN}$

$V_a = 240 + 300 - 250 = \boxed{290KN = V_a}$

Beam moment at C of girder $M_c = 290 \times 60 - 240 \times 35$

$M_c = 9000\ KNm$

Beam moment at 240KN = $290 \times 25 = 7250 KNm$

Beam moment at 300KN = $250 \times 40 = 10000\ KNm$

Now the Horizontal reaction at each of cable

$$H = \frac{M_c}{h} = \frac{9000}{12} = 750KN$$

WKT

$$H = \frac{w_e l^2}{8h} = \frac{w_e \times 120^2}{8 \times 12} = 750 \Rightarrow \boxed{w_e = 5KNm}$$

V = Vertical reaction = $\frac{w_e l}{2} = \frac{5 \times 120}{2} = 300KN$

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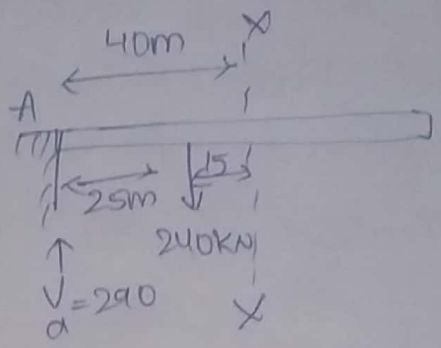
$T_{max} = \sqrt{V^2 + H^2} = \sqrt{300^2 + 750^2} = 807.8\ KN$

∴ The Actual shear force at any section of girder

S = Beam shear − $H \cdot \tan\theta$

Beam shear @ 40m from left end

$= 290 - 240 = 50\ KN$



For cable @ any point

$$\tan\theta = \frac{4h}{l^2}(l - 2x)$$

$$= \frac{4 \times 12}{120^2}(120 - 2 \times 40) = \frac{2}{15}$$

(A) Now The Actual S.F @ 40m from left $= 50 - 750 \times \frac{2}{15}$

$= -50\ KN$

The B.M at any section for girder

M = Beam moment − H moment $= M_{Beam} - H \times y$

Beam Moment @ 40m from left.

$= 290 \times 40 - 240 \times 15 = 8000\ KNm$

$$y = \frac{4h}{l^2} x(l - x) = \frac{4 \times 12}{120^2} \times 40 \times (120 - 40) = \frac{32}{3}\ m$$

Actual B.M $M = 8000 - 750 \times \frac{32}{3}$ ~~= 7500 KNm~~ 0

Beam Moment Diagram

