TEEGALA KRISHNA REDDY ENGINEERING COLLEGE

(UGC-Autonomous)

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DEPARTMENT OF CIVIL ENGINEERING
LECTURE NOTES

ACADEMIC YEAR: 2023-24

REGULATION: R20

YEAR/SEMESTER: III - I

SUBJECT: **STRUCTURALANALYSIS– II**

SUBJECT CODE: **20CE5PC04**

FACULTY NAME: P.CHAITANYA

SYLLABUS:

UNIT – III

Approximate Methods of Analysis: Introduction –Analysis of multi-storey frames for lateral loads. Portal Method, Cantilever method and Factor method – Analysis of multi-storey frames for gravity loads – Substitute Frame method – Analysis of Mill bents.

Unit- III. Dt:- 09/11/22

Analysis of Multi-Story Frames

A building frame may Contain any no: of bays and Storage. Therefore a multi-Storied, multi-bay frame is Complicated Statically indeterminate Structure.

a) Vertical loads – Dead load, Live load

b) Lateral / Horizontal loads – Seismic, wind loads

Methods of Analysis of Lateral loads:

(i) Portal method

(ii) Cantilever method

(iii) Factor method.

(i) PORTAL METHOD

→ This method is most Suitable when the HEIGHT of the building less than that of the WIDTH of the building.

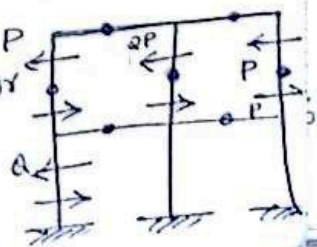
H < L

→ It is Proposed by SMITH. (Also Called Smith method).

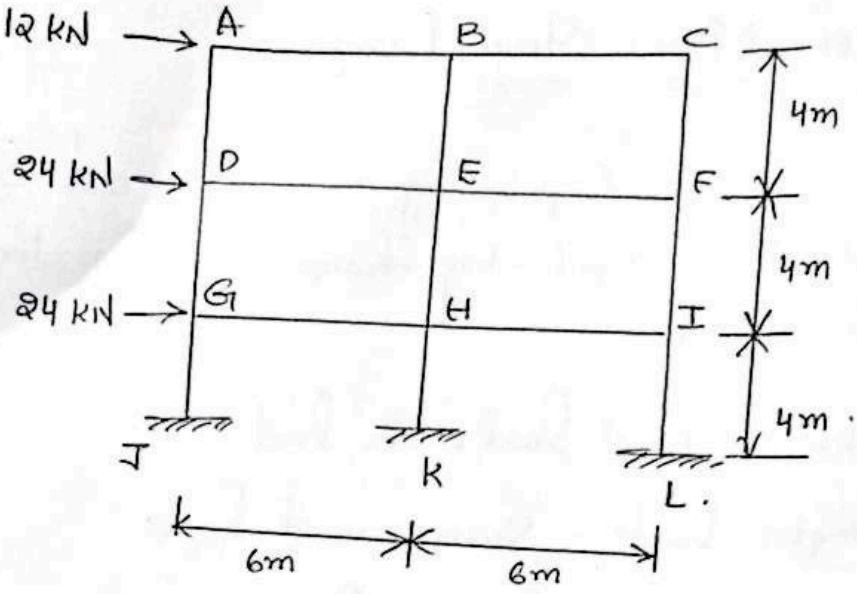
Assumptions:-

→ The Point of Contraflexure in all the members that is beams, Columns lies at these mid Points.

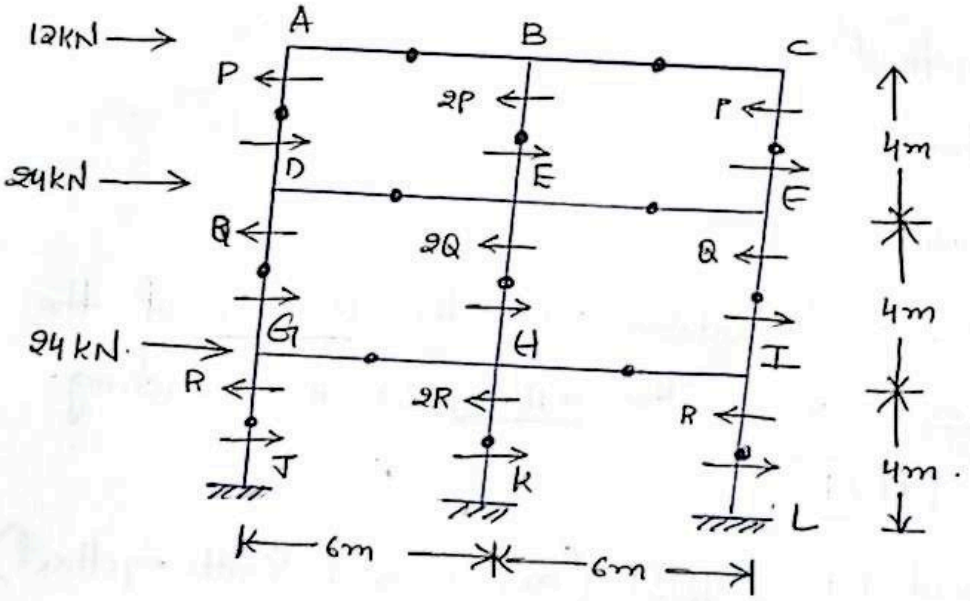
→ The Horizontal Shear taken by Each interior Column is twice to that of taken by Each Exterior Column.



Q:- Analysis of building frames by Portal method:-



Sol:- Consider the two assumptions and draw the frame:-



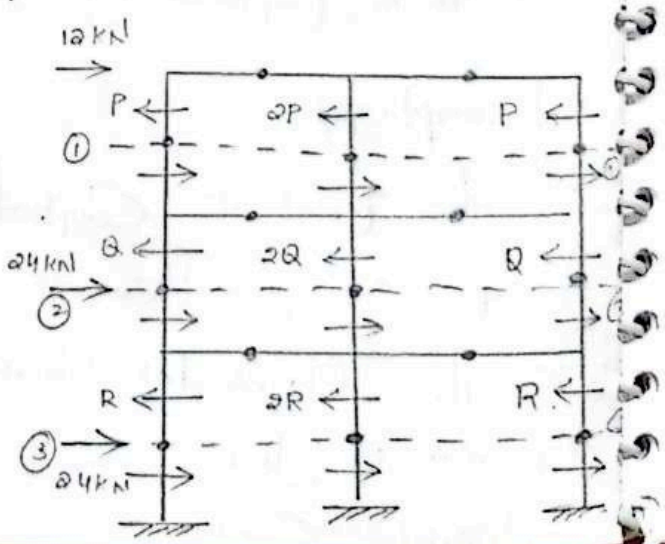
Step ①:- Horizontal Shear

Consider Section ①-①

$P + 2P + P = 12$.

$\boxed{P = 3\ kN}$

Consider Section ②-②



$Q + 2Q + Q = 12 + 24$

$\boxed{Q = 9\,kN}$

Consider Section ③ - ③

$R + 2R + R = 24 + 12 + 24$

$\boxed{R = 15\,kN}$

Step ② :- Calculation of Bending moments in Columns (M).

$M_{AD} = M_{DA} = M_{CF} = M_{FC} = P \times \frac{h}{2} = 3 \times \frac{4}{2} = 6\ kN\cdot m.$

$M_{BE} = M_{EB} = 2P \times \frac{h}{2} = 2 \times 3 \times \frac{4}{2} = 12\ kN\cdot m.$

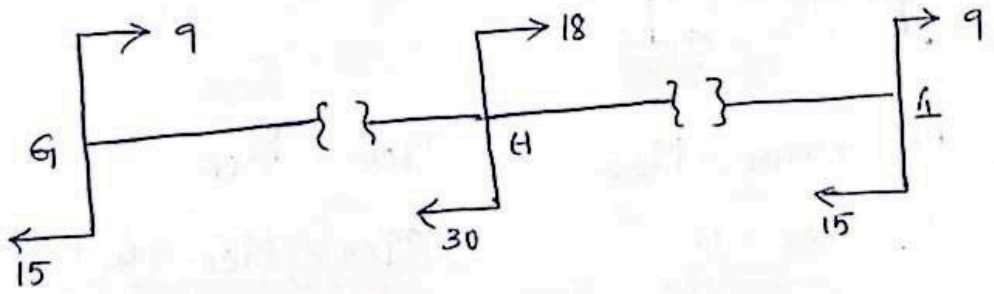
$M_{DG} = M_{GD} = M_{FI} = M_{IF} = Q \times \frac{h}{2} = 9 \times \frac{4}{2} = 18\ kN\cdot m.$

$M_{EH} = M_{HE} = 2Q \times \frac{h}{2} = 2 \times 9 \times \frac{4}{2} = 36\ kN\cdot m.$

$M_{GJ} = M_{JG} = M_{IL} = M_{LI} = R \times \frac{h}{2} = 15 \times \frac{4}{2} = 30\ kN\cdot m.$

$M_{HK} = M_{KH} = 2R \times \frac{h}{2} = 2 \times 15 \times \frac{4}{2} = 60\ kN\cdot m.$

Step ③ : BMD in Beams (M)



Joint G :

$M_{GH} = M_{GD} + M_{GJ} = 18 + 30 = 48\ kN\cdot m.$

Joint H :

$M_{HG} + M_{HI} = M_{HE} + M_{HK}.$

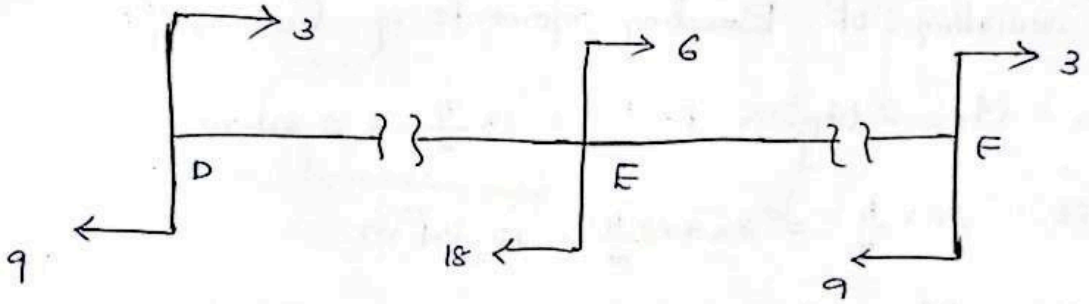
$48 + m_{HI} = 36 + 60$

$m_{HI} = 48$ kN.m.

Joint I:-

$m_{IH} = M_{IE} + M_{IL}$

$m_{IH} = 18 + 30 = 48$ kN-m.



Joint D:-

$m_{DE} = M_{DA} + M_{DG}$

$= 6 + 18$

$= 24$ kN-m.

Joint E

$m_{ED} + m_{EF} = M_{EB} + M_{EH}$

$24 + m_{EF} = 12 + 36$

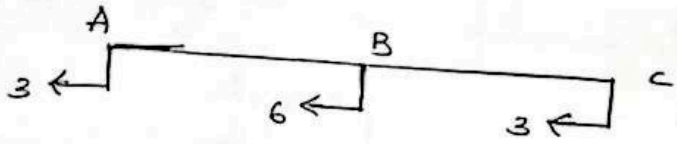
$m_{EF} = 24$ kN-m.

Joint F:-

$m_{FE} = M_{FC} + M_{FI}$

$m_{FE} = 6 + 18$

$m_{FE} = 24$ kN-m



$m_{AB} = M_{AD} = 6$, $m_{BA} + m_{BC} = M_{BE}$

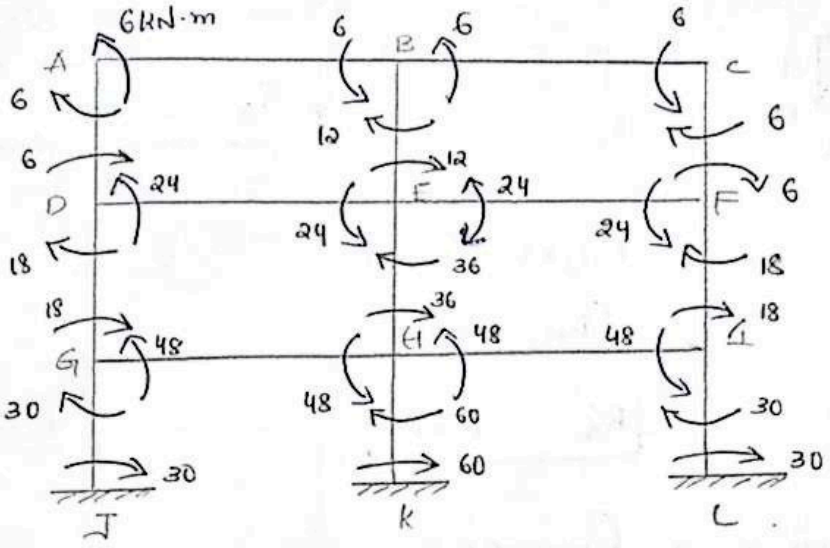
$6 + m_{BC} = 12$

$m_{BC} = 6$ kN.m.

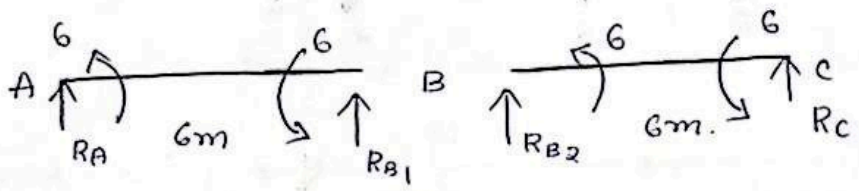
$m_{CB} = M_{CF}$

$m_{CB} = M_{CF} = 6$ kN-m.

Step (4): Bending moment in Beams (m) & Columns (M)



Step (5) :- Shear forces in Beams :-



Clockwise moments = Anticlockwise moments.

$R_A \times 6 = 6+6$

$R_A = 2\,kN$

$R_A + R_{B_1} = 0\ kN$

$2 + R_{B_1} = 0$

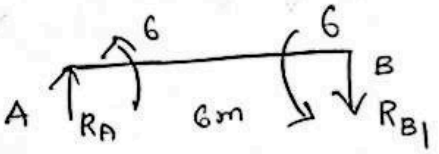
$R_{B_1} = -2\,kN$

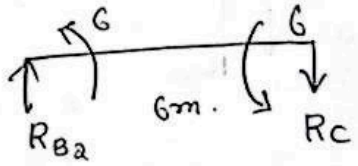
$R_{B_2} \times 6 = 6+6$

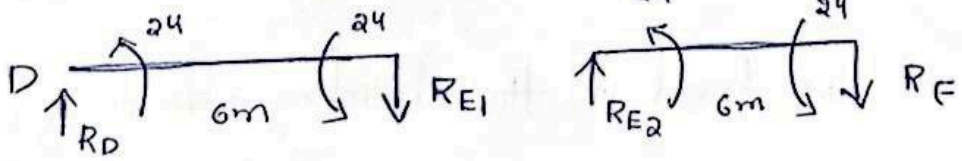
$R_{B_2} = 2\,kN$

$R_{B_2} + R_C = 0$

$R_C = -2\,kN$







$R_D \times 6 = 24+24$

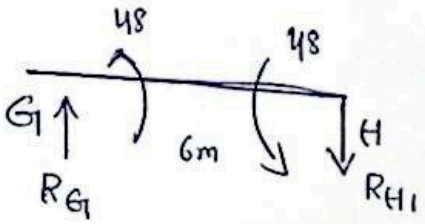
$R_D = 8\,kN$

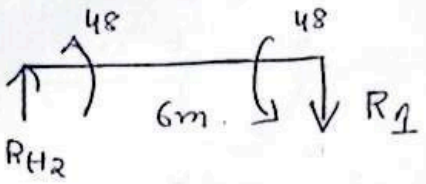
$R_{E_1} = -8\,kN$

$R_{E_2} \times 6 = 24+24$

$R_{E_2} = 8\,kN$

$R_F = -8\,kN$





$R_{G1} \times 6 = 48 + 48$

$R_{G1} = 16\ kN$

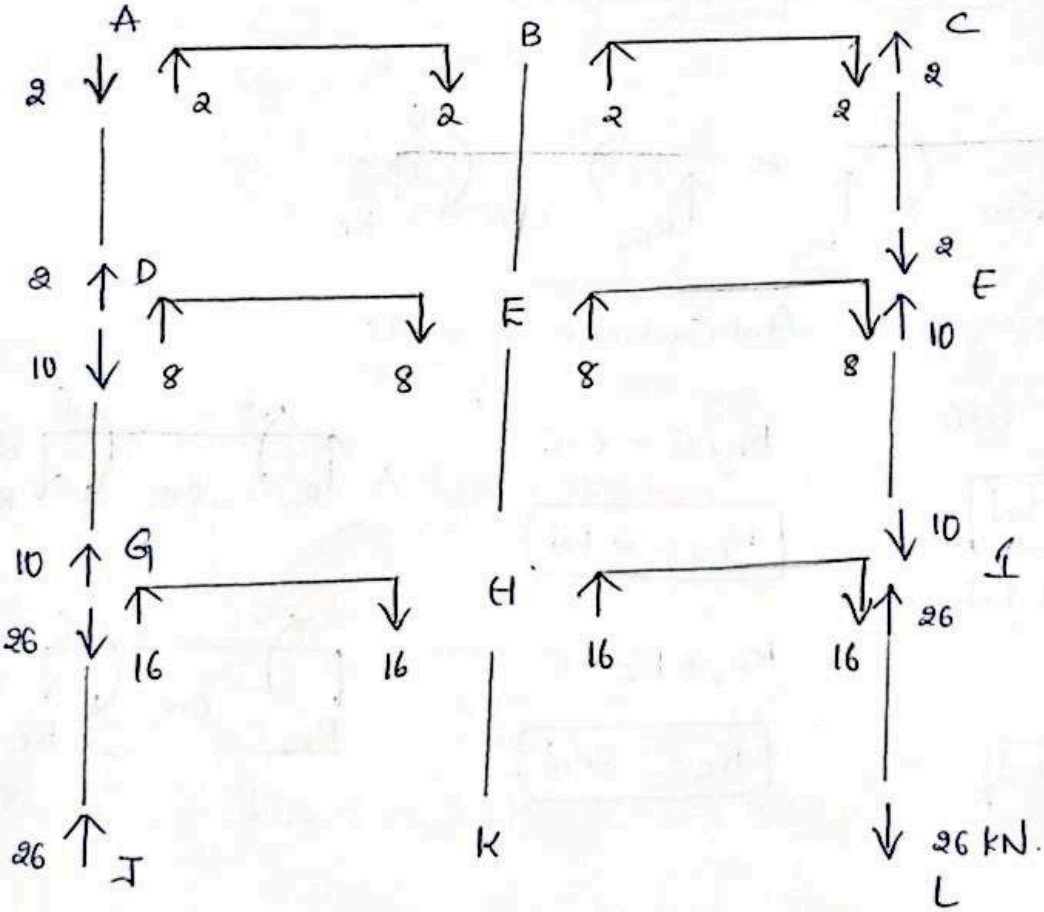
$R_{H1} = -16\ kN$

$R_{H2} \times 6 = 48 + 48$

$R_{H2} = 16\ kN$

$R_{I1} = -16\ kN$

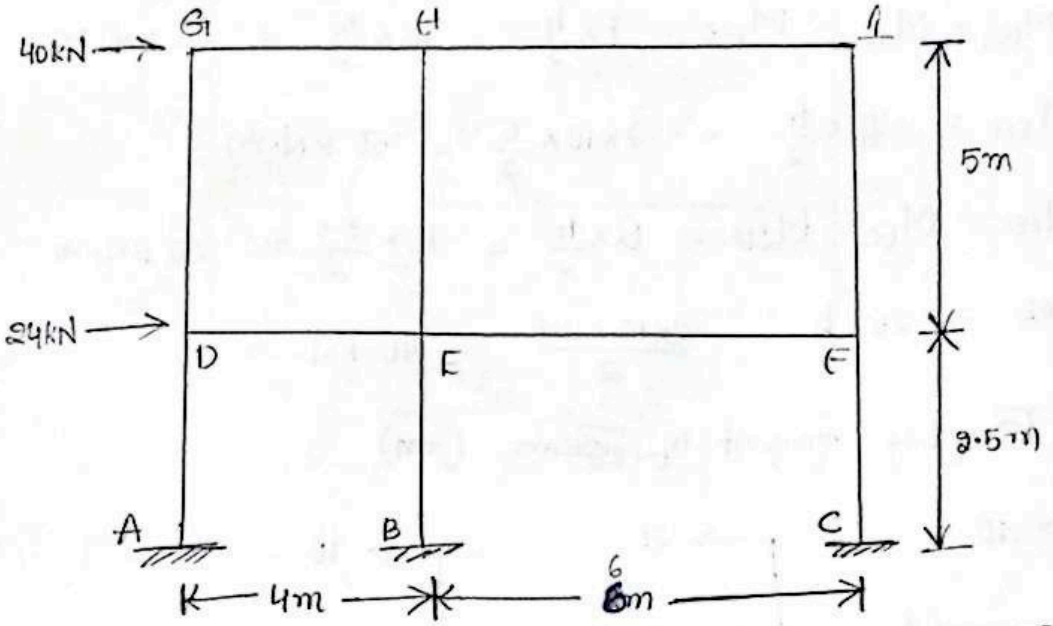
Step ⑥: Axial forces in Columns :-



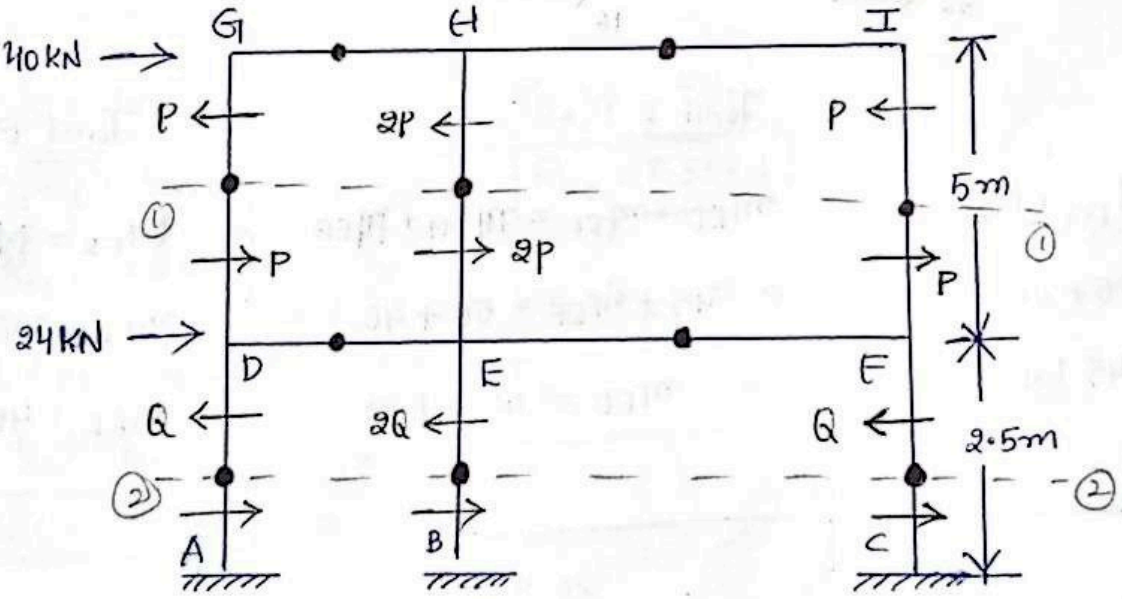
Net Axial force at the based of the building frame

$26 - 26 = 0$

Q:- Analysis of building frames by Portal method:-



Sol:- Consider the two assumptions and draw the frame:-



Step ①:- Horizontal Shear:

Consider Section ①-①

$P + 2P + P = 40\ kN$

$4P = 40\ kN$

$\boxed{P = 10\ kN}$

Consider Section ②-②

$Q + 2Q + Q = 24 + 40$

$4Q = 64\ kN$

$\boxed{Q = 16\ kN}$

Step ② :- Calculating of Bending moments in Columns (M).

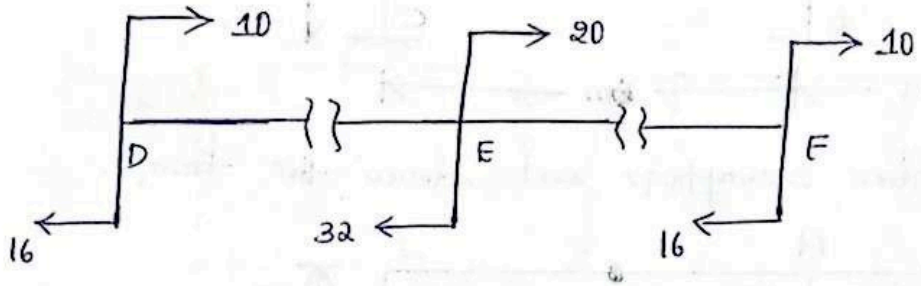
$M_{GD} = M_{DG} = M_{IF} = M_{FI} = P \times \frac{h}{2} = 10 \times \frac{5}{2} = 25\ kN\cdot m$

$M_{HE} = M_{EH} = 2P \times \frac{h}{2} = 2 \times 10 \times \frac{5}{2} = 50\ kN\cdot m$

$M_{DA} = M_{AD} = M_{FC} = M_{CF} = Q \times \frac{h}{2} = 16 \times \frac{2.5}{2} = 20\ kN\cdot m$

$M_{EB} = M_{BE} = 2Q \times \frac{h}{2} = \frac{2 \times 16 \times 2.5}{2} = 40\ kN\cdot m$

Step ③ :- Bending moment in Beams (m).



Joint D :-

$m_{DE} = M_{DG} + M_{DA}$.

$= 25 + 20$

$= 45\ kN\cdot m$.

Joint E :-

$m_{ED} + m_{EF} = m_{EH} + m_{EB}$

$\Rightarrow 45 + m_{EF} = 50 + 40$

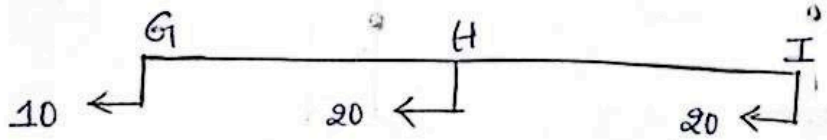
$m_{EF} = 45\ kN\cdot m$

Joint F

$m_{FE} = M_{FI} + M_{FC}$

$m_{FE} = 25 + 20$

$m_{FE} = 45\ kN\cdot m$



$m_{GH} = M_{GD}$

$= 25\ kN\cdot m$

$m_{HG} + m_{HI} = M_{HE}$

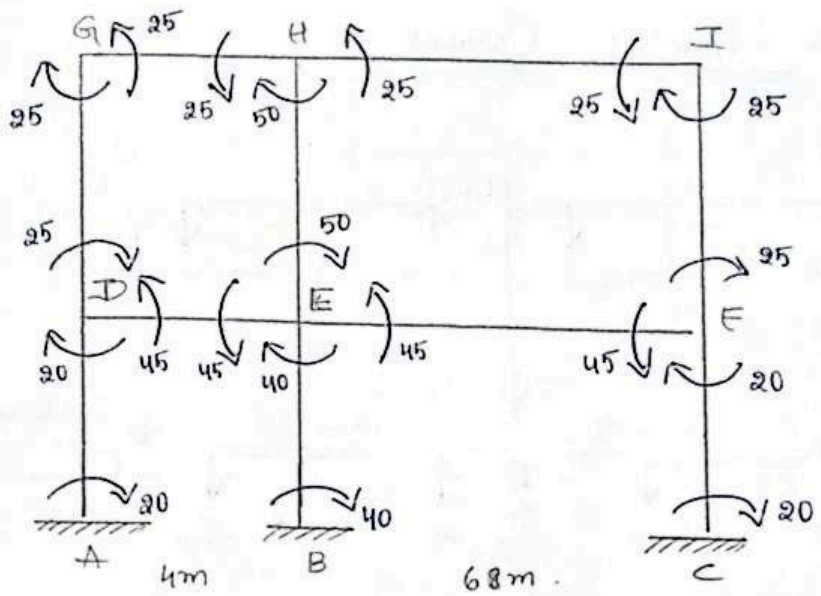
$25 + m_{HI} = 50$

$m_{HI} = 25\ kN\cdot m$.

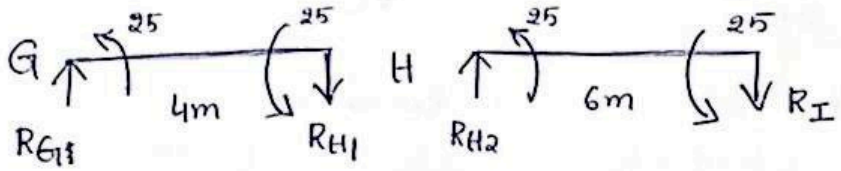
$m_{IH} = M_{IF}$

$= 25\ kN\cdot m$.

Step ④ :- Bending moments in Beams and Columns :- (M)
(m)



Step (5) :- Shear forces in beams.



C.W = ACW

$R_G \times 4 = 25 + 25$

$\boxed{R_G = 12.5\ kN}$

$R_G + R_{H_1} = 0$

$12.5 + R_{H_1} = 0$

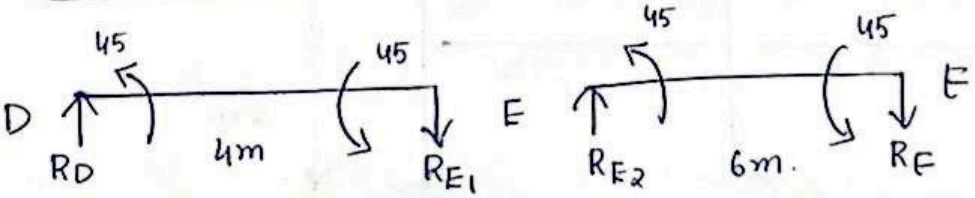
$\boxed{R_{H_1} = -12.5\ kN}$

$R_{H_2} \times 6 = 50$

$\boxed{R_{H_2} = 8.33\ kN}$

$R_{H_2} + R_I = 0$

$\boxed{R_I = -8.33\ kN}$



CW = ACW

$R_D \times 4 = 90\ kN$

$\boxed{R_D = 22.5\ kN}$

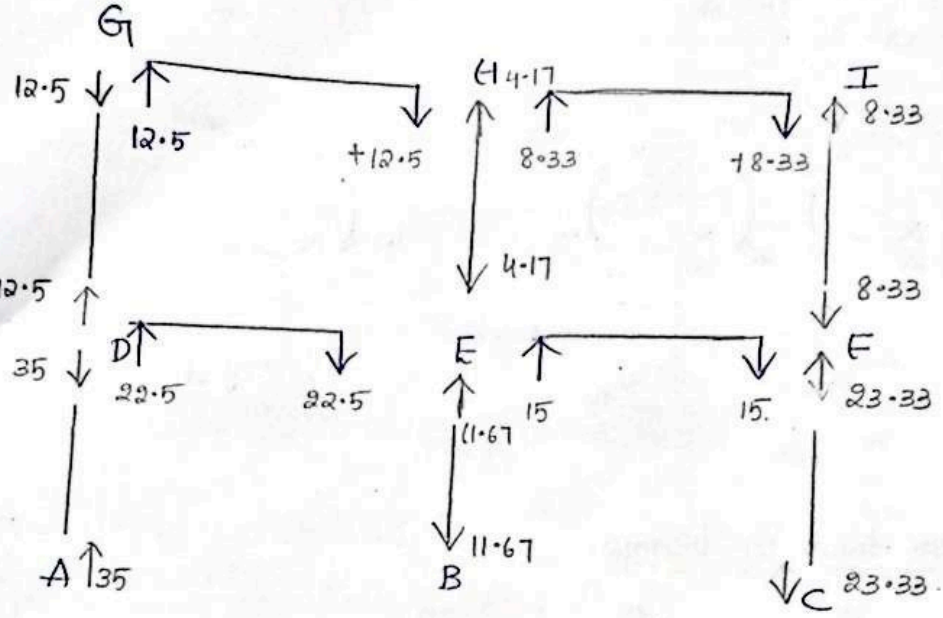
$\boxed{R_{E_1} = -22.5\ kN}$

$R_{E_2} \times 6 = 90\ kN$

$\boxed{R_{E_2} = 15\ kN}$

$\boxed{R_F = -15\ kN}$

Step ⑥ : Axial forces in Columns

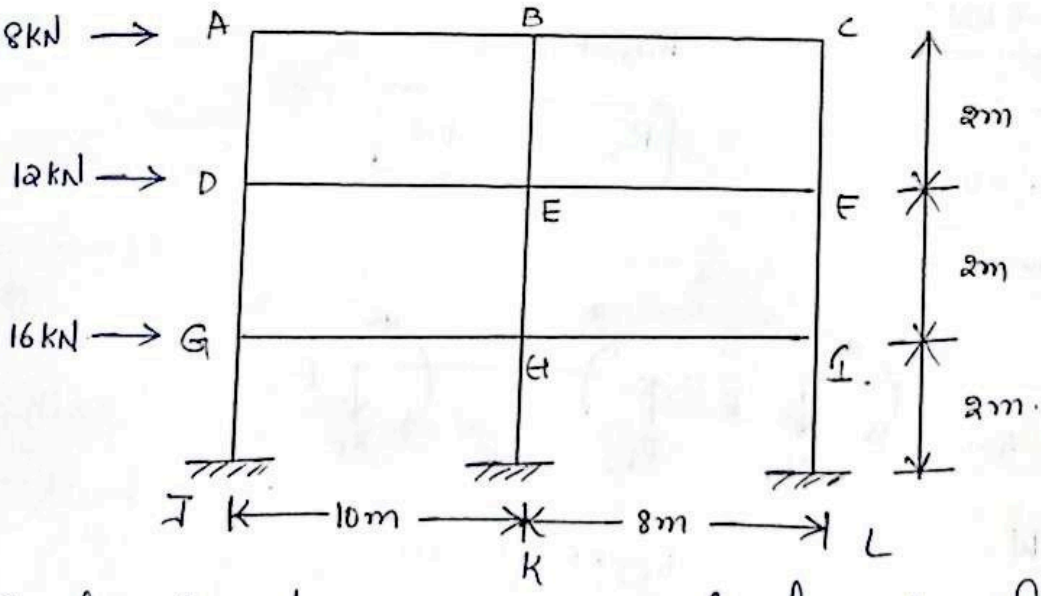


Net Axial force at the based on the building frame

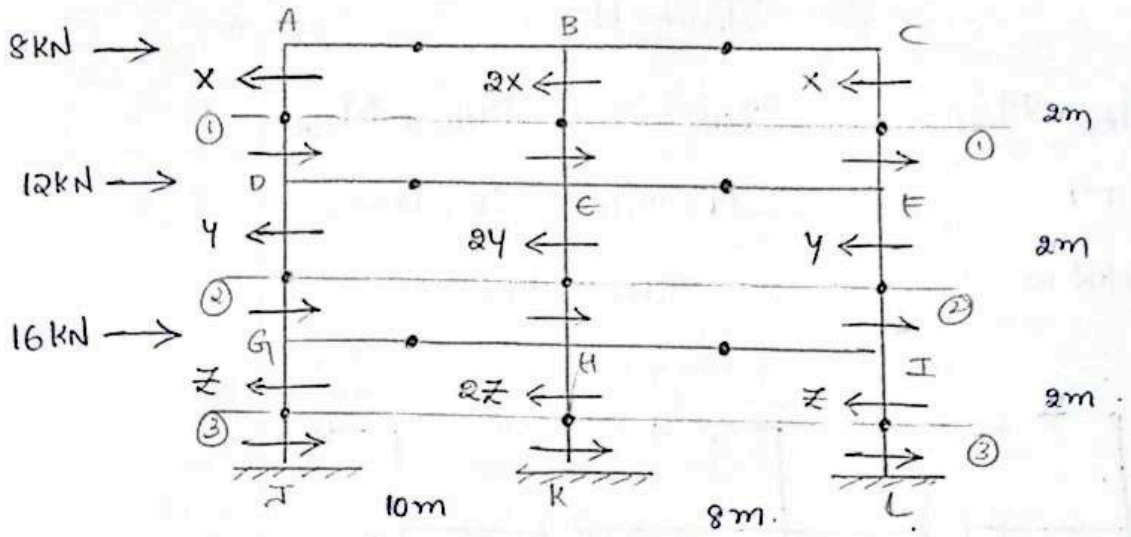
$= 35 - 23.33 - 11.67 = 0.$

Q:- Analysis of building frames by Portal method :-

Extra



Sol:- Consider the two assumptions and draw the figure :-



Step ①:- Horizontal Shear:

$X + 2X + X = 8kN$

$4X = 8kN$

$\boxed{X = 2kN}$

Section ②-②

$Y + 2Y + Y = 12 + 8$

$4Y = 20kN$

$\boxed{Y = 5kN}$

Section ③-③

$Z + 2Z + Z = 36kN$

$4Z = 36kN$

$\boxed{Z = 9kN}$

Step ②:- Calculation of Bending moments in Columns (M)

$M_{AD} = M_{DA} = M_{CF} = M_{FC} = 2 \times \frac{h}{2} = 2 \times \frac{2}{2} = 2\ kN \cdot m.$

$M_{BE} = M_{EB} = 4 \times \frac{h}{2} = 4 \times \frac{2}{2} = 4\ kN \cdot m.$

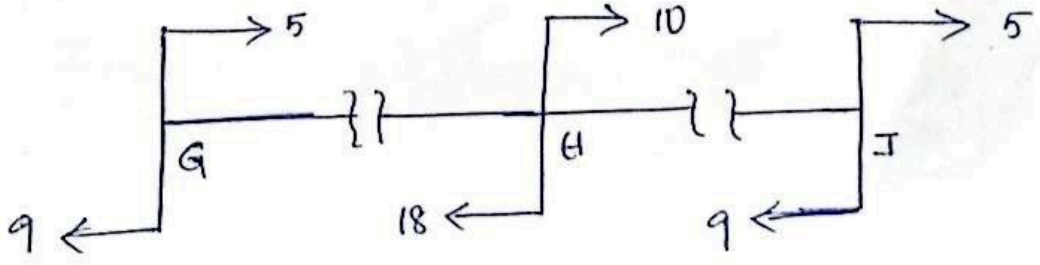
$M_{DG} = M_{GD} = M_{EI} = M_{IF} = Y \times \frac{h}{2} = 5 \times \frac{2}{2} = 5\ kN \cdot m.$

$M_{EH} = M_{HE} = 2Y \times \frac{h}{2} = 2 \times 5 \times \frac{2}{2} = 10\ kN \cdot m.$

$M_{GJ} = M_{JG} = M_{IL} = M_{LI} = Z \times \frac{h}{2} = 9 \times \frac{2}{2} = 9kN \cdot m.$

$M_{HK} = M_{KH} = 2Z \times \frac{h}{2} = 18 \times \frac{2}{2} = 18\ kN \cdot m.$

Step ③:- Calculation of Bending moments in Beams (m).



Joint G:

$m_{GH} = M_{GD} + M_{GI}$

$m_{GH} = 5 + 9$

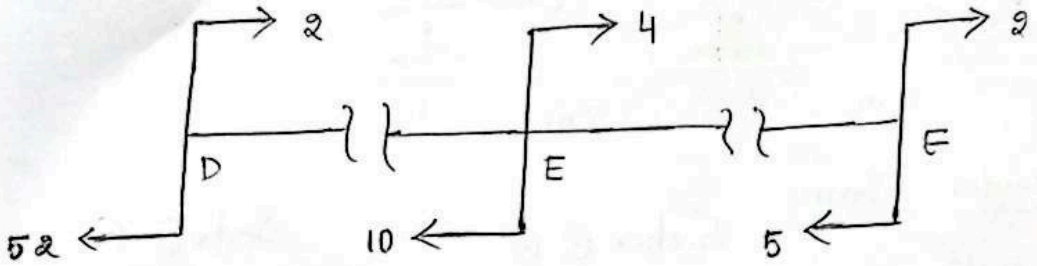
$m_{GH} = 14\ kN.m$.

Joint H :-

$m_{HG} + m_{HI} = M_{HE} + M_{HK}$

$14 + m_{HI} = 10 + 18$

$m_{HI} = 14\ kN.m$.



Joint D :-

$m_{DE} = M_{DA} + M_{DG}$

$2 + 5$

$m_{DE} = 7\ kN.m$.

Joint E:

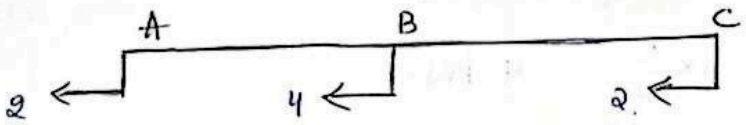
$m_{ED} + m_{EF} = M_{EB} + M_{EH}$

$7 + m_{EF} = 4 + 10$

$m_{EF} = 7\ kN.m$.

Joint F:-

$m_{FE} = 7\ kN.m$.



$m_{AB} = M_{AD} = 2\ kN.m$.

$m_{BA} + m_{BC} = M_{BE}$

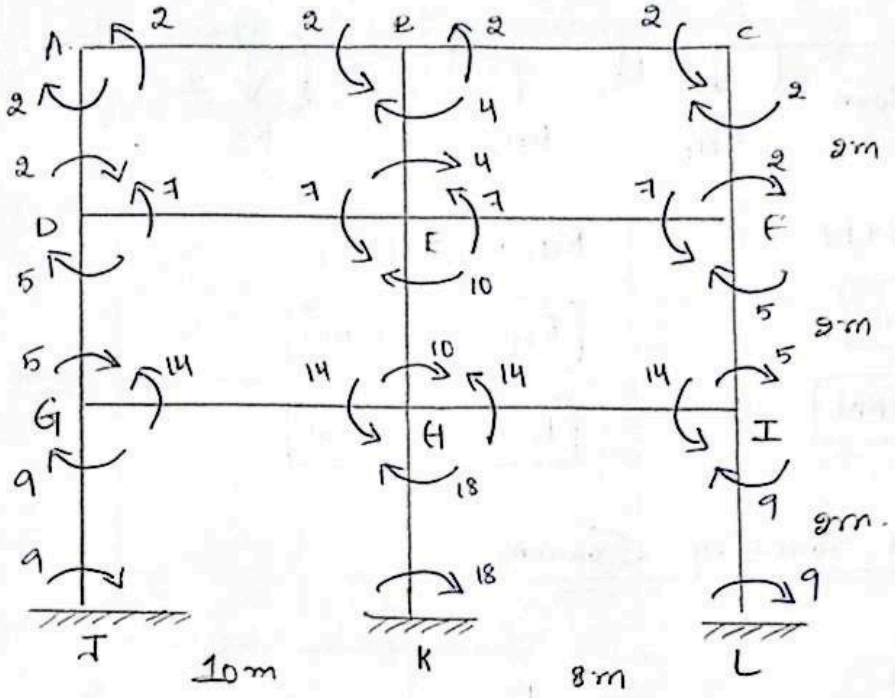
$2 + m_{BC} = 4$

$m_{BC} = 2\ kN.m$

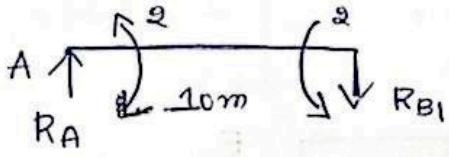
$m_{CB} = M_{CF}$

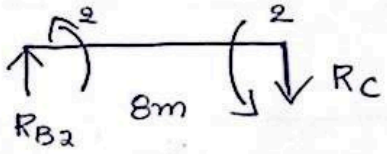
$m_{CB} = 2\ kN.m$.

Step (4) :- Bending moments in Beams and Columns (M)
(m)



Step ⑤ :- Shear forces in Beams





CW = ACW

$R_A \times 10 = 2+2$

$\boxed{R_A = 4/10 = 0.4kN}$

$R_A + R_{B1} = 0$

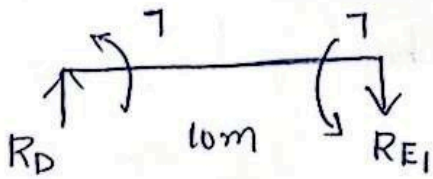
$\boxed{R_{B1} = -0.4kN}$

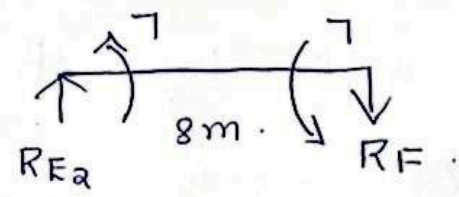
$R_{B_2} \times 8 = 4$

$\boxed{R_{B_2} = 0.5\ kN}$

$R_{B_2} + R_C = 0$

$\boxed{R_C = -0.5\ kN}$





$R_D \times 10 = 14$

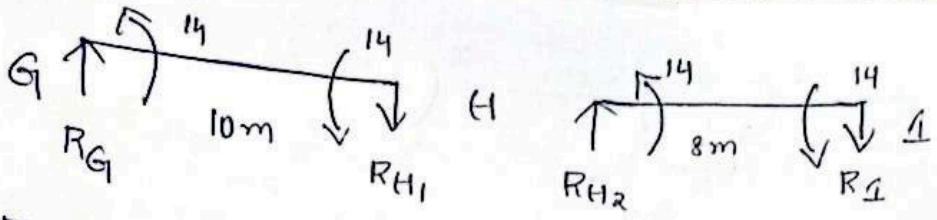
$\boxed{R_D = 1.4kN}$

$\boxed{R_{E1} = -1.4\ kN}$

$R_{E_2} \times 8 = 14$

$\boxed{R_{E_2} = 1.75\ kN}$

$\boxed{R_F = -1.75\ kN}$



$R_G \times 10 = 14+14$

$R_G = 2.8\,kN$

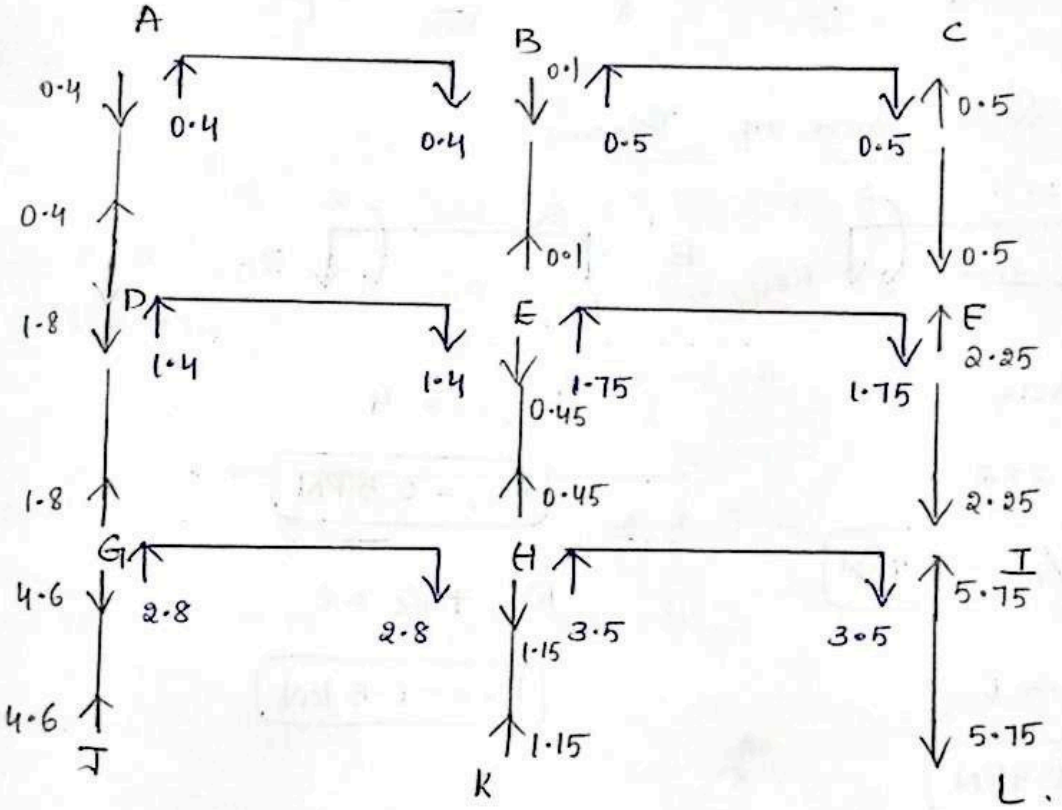
$R_{H1} = -2.8\,kN$

$R_{H2} \times 8 = 14+14$

$R_{H2} = 3.5\,kN$

$R_I = -3.5\,kN$

Step (6): Axial forces in Columns



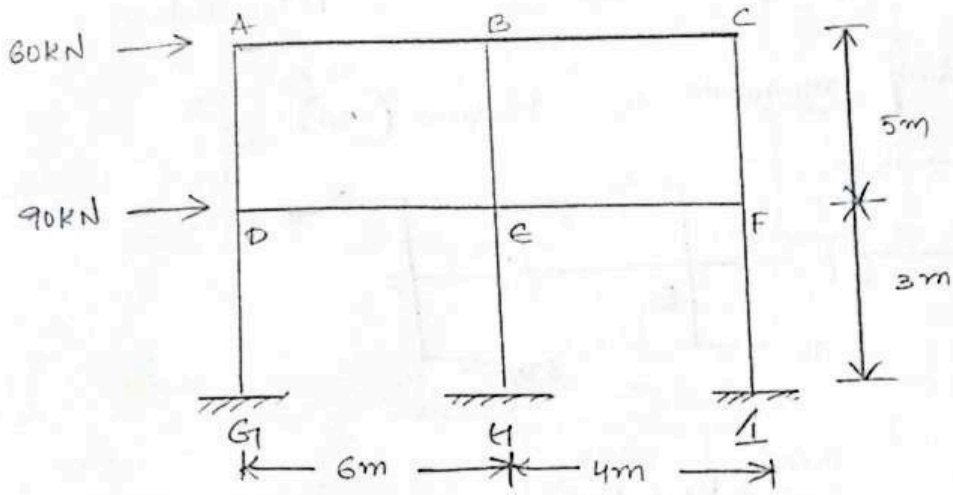
Net Axial force at the based of the building frame.

$4.6 + 1.15 - 5.75$

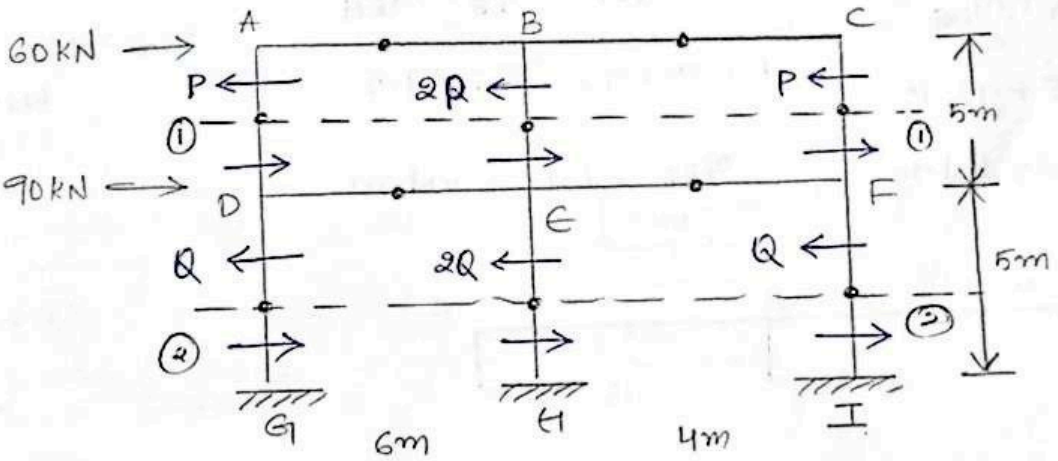
$= 0$

Q:- Analyse the multi-Storey building frame by Portal method or Smith's method:-

Dt:- 10/11/22



Sol:- Consider the two assumptions and Show in figure.



Step ①:- Horizontal Shear:-

Section ①-①

$P + 2P + P = 60$

$4P = 60\ kN$

$\boxed{P = 15\ kN}$

Section ②-②

$Q + 2Q + Q = 60 + 90$

$4Q = 150$

$\boxed{Q = 37.5\ kN}$

Step ②:- Bending moments in Columns (M)

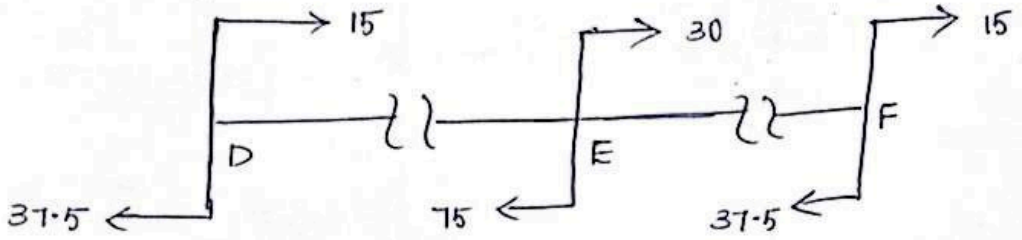
$M_{AD} = M_{DA} = M_{CF} = M_{FC} = P \times \frac{h}{2} = 15 \times \frac{5}{2} = 37.5\ kN$

$M_{BE} = M_{EB} = 2P \times h/2 = 2 \times 15 \times 5/2 = 75\ kN$

$M_{DG} = M_{GD} = M_{EI} = M_{IF} = Q \times \frac{h}{2} = \frac{37.5 \times 3}{2} = 93.75$ kN·m.

$M_{EH} = M_{HE} = 2Q \times \frac{h}{2} = \frac{2 \times 37.5 \times 3}{2} = 187.5$ kN·m.

Step ③ : Bending moments in Beams (m) :-



$m_{DA} + m_{DG} =$

Joint D :-

$m_{DE} = M_{DA} + M_{DG}$

$= 37.5 + 93.75$

$m_{DE} = 131.25$ kN·m.

Joint E :-

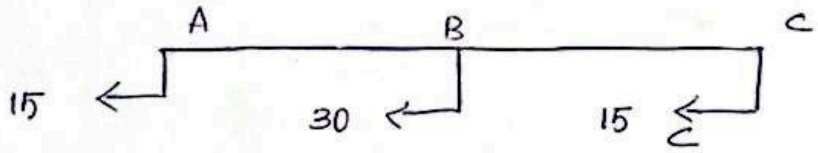
$m_{ED} + m_{EF} = M_{EB} + M_{EH}$

$131.25 + m_{EF} = 75 + 187.5$

$m_{EF} = 131.25$ kN·m.

Joint F

$m_{FE} = 131.25$ kN·m.



Joint A :

$m_{AB} = M_{AD} = 37.5$ kN·m.

Joint B

$m_{BA} + m_{BC} = M_{BE}$

$37.5 + m_{BC} = 75$

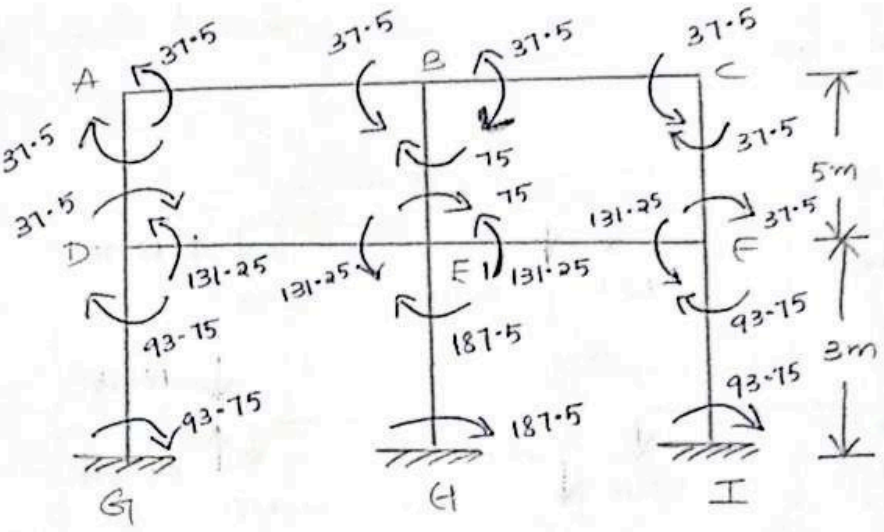
$m_{BC} = 37.5$ kN·m.

Joint C :-

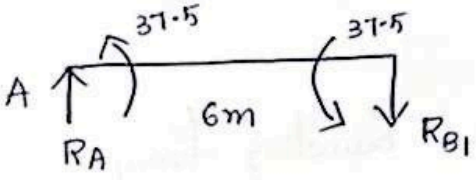
$m_{CB} = M_{CF}$

$= 37.5$ kN·m

Step ④ :- Bending moment in Beams (m) & Columns (M).



Step ⑤ : Shear forces in Beams :-



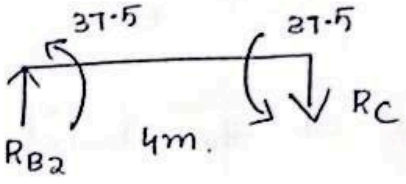
CW = ACW

$R_A \times 6 = 37.5 + 37.5$

$\boxed{R_A = 12.5\ kN}$

$R_A + R_{B1} = 0$

$\therefore \boxed{R_{B1} = -12.5\ kN}$

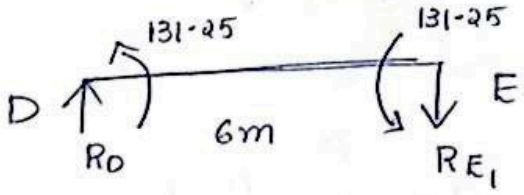


CW = ACW

$R_{B2} \times 4 = 75$

$\boxed{R_{B2} = 18.75\ kN}$

$\boxed{R_C = -18.75\ kN}$

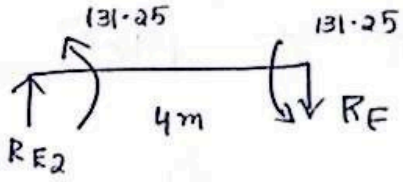


$R_D \times 6 = 131.25 + 131.25$

$\boxed{R_D = 43.75\ kN}$

$R_D + R_{E1} = 0$

$\therefore \boxed{R_{E1} = -43.75\ kN}$

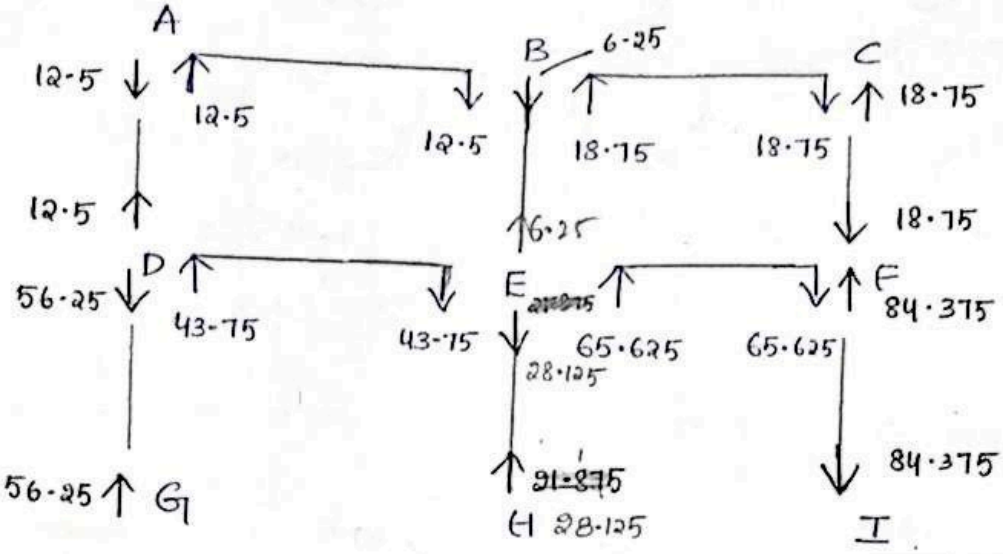


$R_{E2} \times 4 = 131.25 + 131.25$

$\boxed{R_{E2} = 65.625\ kN}$

$\boxed{R_E = -65.625\ kN}$

Step ⑥ :- Axial forces in Columns



Net Axial forces at the based on the building frames

$$56.25 + 28.125 - 84.375$$

$$= 0.$$

CANTILEVER METHOD :-

M:- 23 | 11 | 22

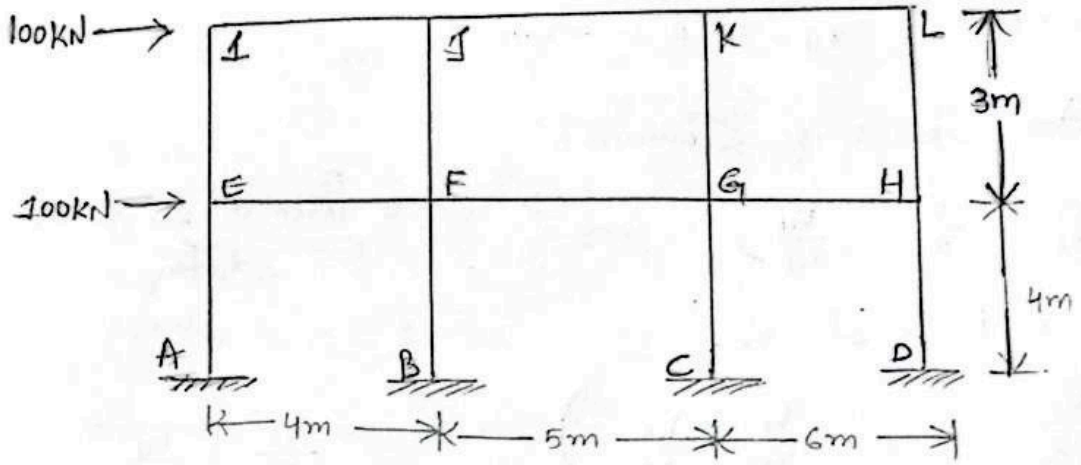
This method was proposed by "Wilson." This method is preferable when the height of the building is greater than the width of the building. H7B

Assumptions :-

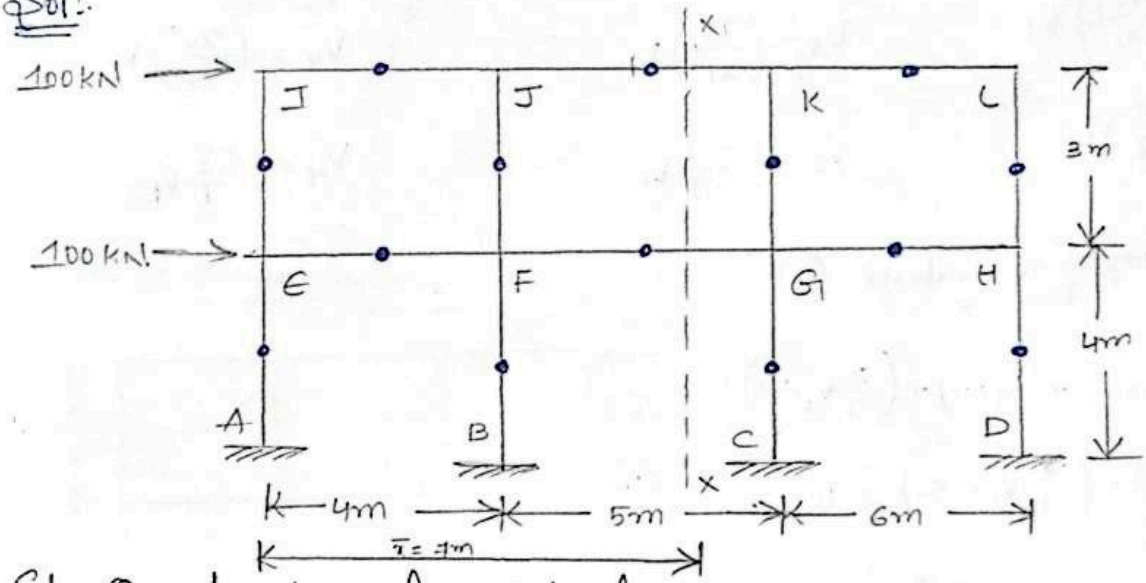
→ point of Contraflexures lies at the mid points of beams & Columns.

→ Axial Stresses in the Columns are directly proportional to these distances from the Centroidal Vertical axis of the building frame.

Q:- Analyse the building frame shown in the figure below by Cantilever method.



Sol:-



Step (1): Location of Centroidal axis

Assume Cross-Sectional areas of all the Columns are Equal and let it be 'a'.

→ Calculation of Centroidal axis distance $\bar{x}$ from the Columns A, E, I.

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + \cdots}{a_1 + a_2 + \cdots}$$

$$\bar{x} = \frac{[(a\times0)+(a\times0)] + [(a\times4)+(a\times4)] + [(a\times9)+(a\times9)] + [(a\times15)+(a\times15)]}{4a+4a}$$

$\bar{x} = 7m$

$x_1 = 7m$

$x_2 = 7 - 4 = 3m$

$x_3 = 7 - 9 = -2m$

$x_4 = 7 - 15 = -8m$

Step ②: Axial Stresses in Columns:

$\frac{V_1}{x_1} = \frac{V_2}{x_2} = \frac{V_3}{x_3} = \frac{V_4}{x_4}$ → Axial force ∝ Centroidal distance.

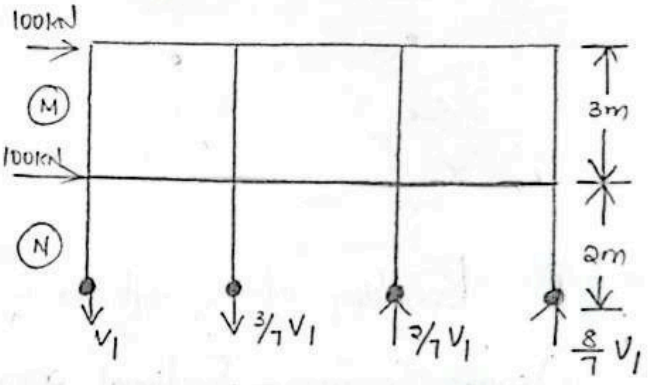
∴ Write in terms of V_1

$\frac{V_1}{x_1} = \frac{V_2}{x_2}$ $\quad \frac{V_1}{x_1} = \frac{V_3}{x_3}$ $\quad \frac{V_1}{x_1} = \frac{V_4}{x_4}$

$\Rightarrow V_2 = \left(\frac{x_2}{x_1}\right) V_1$ $\quad V_3 = \left(\frac{x_3}{x_1}\right) V_1$ $\quad V_4 = \left(\frac{x_4}{x_1}\right) V_1$

$V_2 = \frac{3}{7} V_1$ $\quad V_3 = \frac{-2}{7} V_1$ $\quad V_4 = \frac{-8}{7} V_1$

= Taking moments about (N)



$(100 \times 5) + (100 \times 2) + (V_1 \times 0) + \left(\frac{-3}{7} V_1 \times 4\right)$

$- \left(\frac{2}{7} V_1 \times 9\right) - \left(\frac{8}{7} V_1 \times 15\right) = 0$

$V_1 = 38.88$ kN

$\therefore V_2 = \left(\frac{3}{7} V_1\right) = 16.66$ kN

$V_3 = \left(\frac{-2}{7} V_1\right) = -11.10$ kN

$V_4 = \left(\frac{-8}{7} V_1\right) = -44.43$ kN.

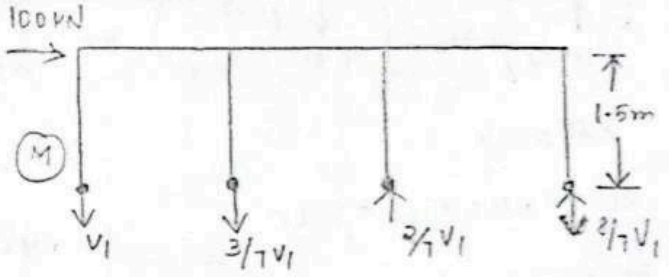
Net axial forces at the based on the building frame

$= V_1 + V_2 + V_3 + V_4$

$= 38.88 + 16.66 - 11.10 - 44.43$

0

Taking the moments about (M).



$(100 \times 1.5) + (V_1 \times 0) + (3/7 V_1 \times 4) - (2/7 V_1 \times 9) - (8/7 V_1 \times 15) = 0$

$V_1 = 8.33$ kN

$V_2 = (3/7 V_1) = 3.57$ kN

$V_4 = (-8/7 V_1) = -9.52$ kN

$V_3 = (-2/7 \times V_1) = -2.38$ kN.

Net axial forces based on the building frame :-

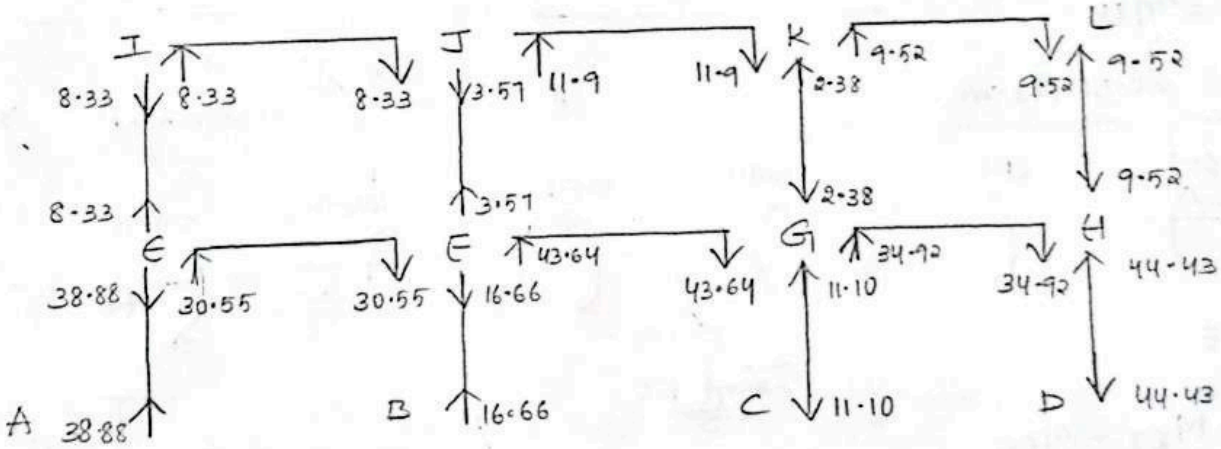
$V_1 + V_2 + V_3 + V_4$

$= 8.33 + 3.57 - 2.38 - 9.52$

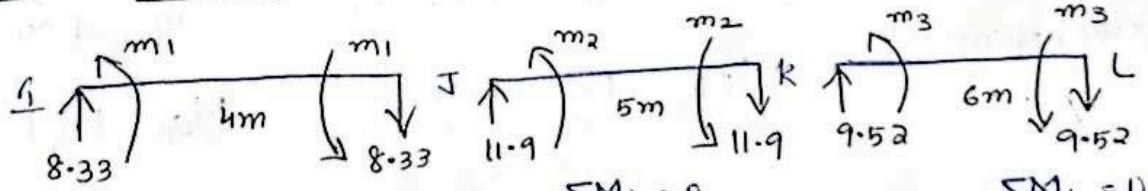
$= 0$.

(V = axial force as per 2nd assumption).

Step (3) :- Shear force in Beams.



Step (4): Bending moments in beams (m) :-



$\Sigma M_J = 0$

CW = ACW

$8.33 \times 4 = m_1 + m_1$ (IJ)

$m_1 = 16.66$ kN.m

$\Sigma M_K = 0$

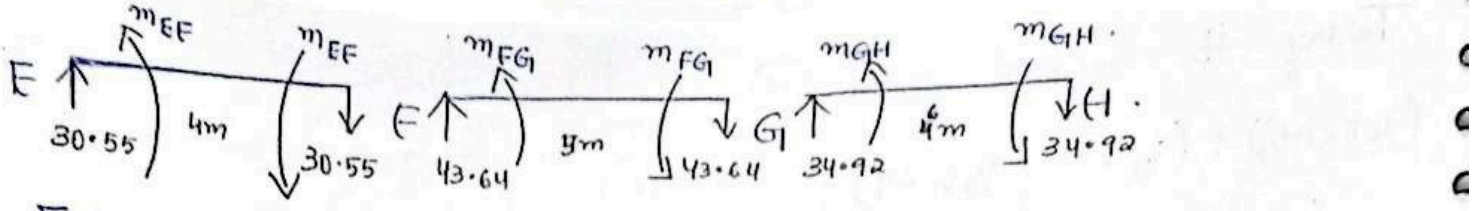
$11.9 \times 5 = m_2 + m_2$

$m_2 = 29.75$ kN.m (JK)

$\Sigma M_L = 0$

$9.52 \times 6 = m_3 + m_3$

$m_3 = 28.56$ kN.m (KL)



$\Sigma M_E = 0$

$30.55 \times 4 = m_{EE} + m_{EE}$

$30.55 \times 4 = 2 m_{EE}$

$m_{EE} = 61.1 \text{ kN·m}$

$\Sigma M_{G_1} = 0$

$43.64 \times 5 = 2 m_{FG_1}$

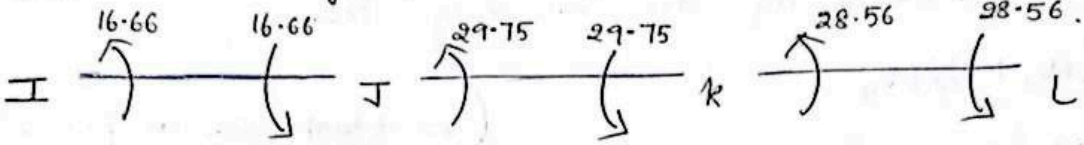
$m_{FG_1} = 109.1 \text{ kN·m}$

$\Sigma M_H = 0$

$34.92 \times 6 = 2 m_{GH}$

$m_{GH} = 104.76 \text{ kN·m}$

Step ⑤ :- Bending moment in Columns (M).



Joint I:

$M_{IE} = m_{IJ}$

$= 16.66 \text{ kN·m}$

$M_{LH} = m_{LK}$

$= 28.56 \text{ kN·m}$

Joint J:

$M_{JF} = m_{JI} + m_{JK}$

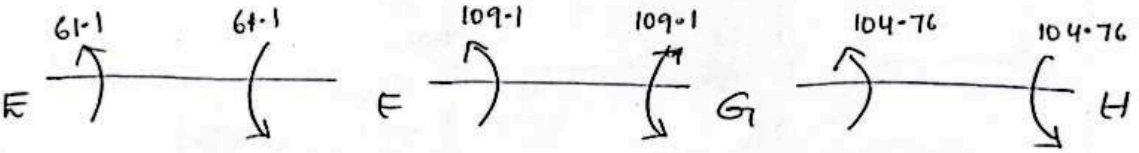
$m_{JF} = 16.66 + 29.75$

$M_{JF} = 46.41 \text{ kN·m}$

$m_{KG_1} = M_{KJ} + M_{KL}$

$m_{KG_1} = 29.75 + 28.56$

$M_{KG} = 58.31 \text{ kN·m}$



Joint E:

$M_{EA} + M_{EI} = m_{EE}$

$M_{EA} = 61.1 - 16.66$

$M_{EA} = 44.44 \text{ kN·m}$

Joint F:

$M_{FB} + M_{FJ} = m_{FE} + m_{FG_1}$

$M_{FB} + 46.41 = 61.1 + 109.1$

$M_{FB} = 123.79 \text{ kN·m}$

Joint G:

$M_{GK} + M_{GF} = m_{GF} + m_{GH}$

$M_{GK} = 109.1 + 104.76 - 58.31$

$M_{GK} = 155.55 \text{ kN·m}$

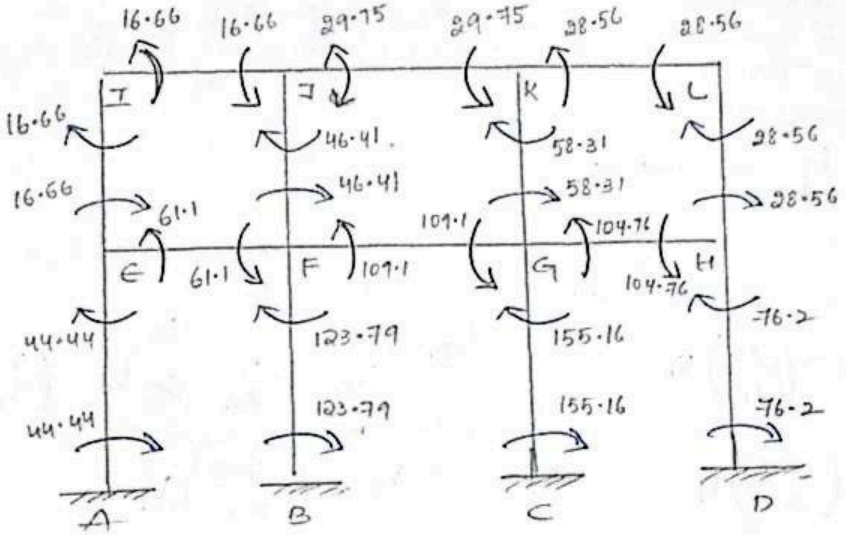
Joint H:

$M_{HL} + M_{HG_1} = m_{HG_1}$

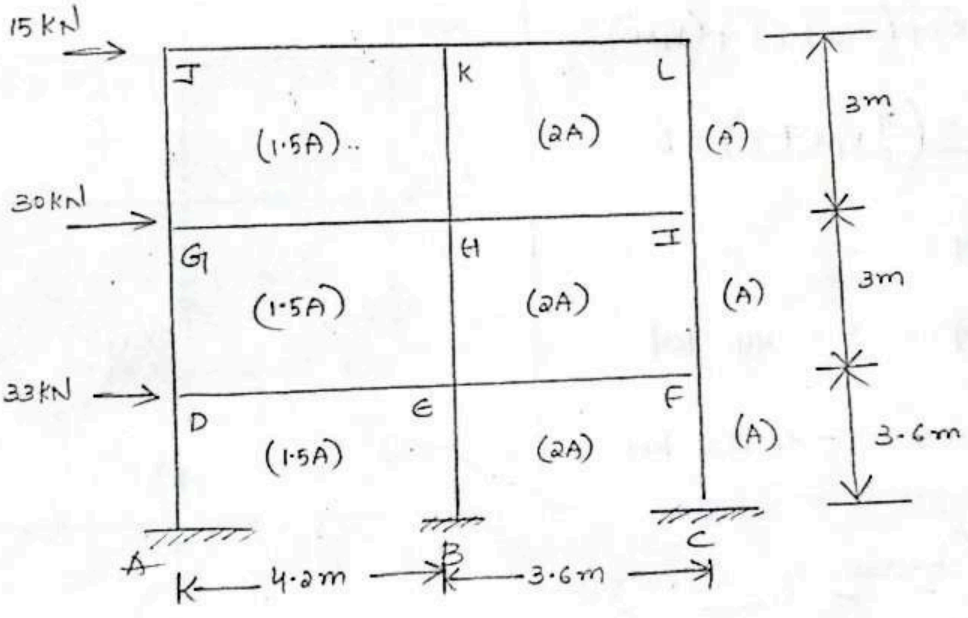
$28.56 + M_{HG_1} = 104.76$

$M_{HG_1} = 76.2 \text{ kN·m}$

Step ⑥: BM's in Beams & Columns.



Q:- Analyse the building frame shown in the figure below by Cantilever method:-

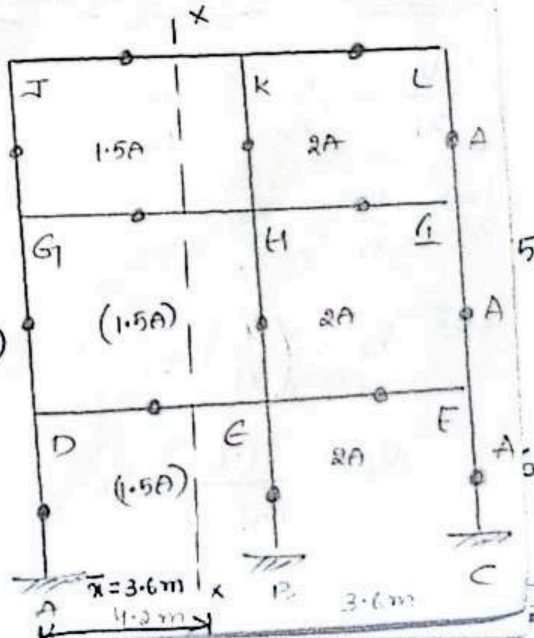


Sol:- Step ①:- Location of Centroidal axis:-

Calculation of Centroidal axis distance $\bar{x}$ from the Columns 'ADGI'.

$$\bar{x} = \frac{(2A \times 4.2) + (2A \times 4.2) + (2A \times 4.2) + (A \times 7.8) + (A \times 7.8) + (A \times 7.8)}{(2 \times 1.5A) + (3 \times 2A) + (3 \times A)}$$

$$\bar{x} = 3.6 \text{ m} //$$



$\bar{x} = 3.6\,m$

$x_1 = 3.6\,m$

$x_2 = 3.6 - 4.2$

$= -0.6\,m$

$x_3 = 3.6 - 7.8$

$= -4.2\,m$

Step ②:- Axial Stresses in Columns.

$$\frac{V_1}{x_1} = \frac{V_2}{x_2} = \frac{V_3}{x_3}$$

$$\Rightarrow \frac{V_1}{x_1} = \frac{V_2}{x_2} \Rightarrow V_2 = \left(\frac{x_2}{x_1}\right) V_1$$

$$= V_2 = \left(\frac{-0.6}{3.6}\right) V_1$$

$V_2 = -0.167\,V_1$ Wrong

Taking moments about 'P'

$$(15 \times 7.8) + (30 \times 4.8) + (33 \times 1.8) + (V_1 \times 0) - \left(\tfrac{1}{6} V_1 \times 4.2\right) - \left(\tfrac{7}{6} V_1 \times 7.8\right) = 0$$

$V_1 = 32.69$

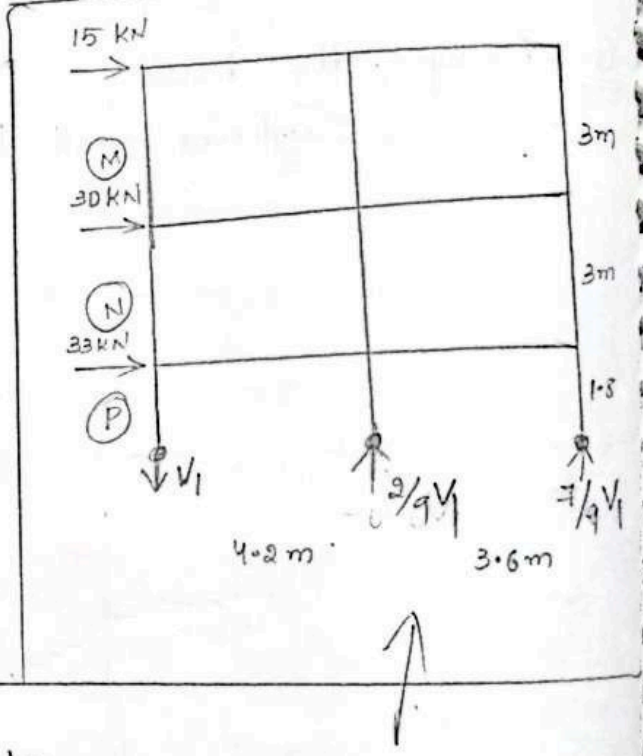
$V_2 = \left(\frac{-0.6}{3.6}\right) \times 32.69 = -5.44$ kN

$V_3 = \left(\frac{-4.2}{3.6}\right) \times 32.69 = -38.13$ kN.

$$\frac{V_1}{x_1} = \frac{V_3}{x_3}$$

$$\Rightarrow \frac{V_1}{x_1} = \frac{V_3}{x_3} \Rightarrow V_3 = \left(\frac{x_3}{x_1}\right) V_1$$

$$V_3 = \left(\frac{-4.2}{3.6}\right) V_1$$



Step ②:- Axial Stresses in Columns:-

$$\frac{V_1}{x_1} = \frac{V_2}{x_2} = \frac{V_3}{x_3}$$

$$\Rightarrow \frac{V_1}{a_1 x_1} = \frac{V_2}{a_2 x_2}$$

$$\Rightarrow V_2 = \frac{a_1}{a_1}\left(\frac{x_2}{x_1}\right) \times V_1$$

$$V_2 = \frac{2(-0.6)}{1.5(3.6)} V_1$$

$V_2 = -2/9\,V_1$

$$\frac{V_1}{a_1 x_1} = \frac{V_3}{a_3 x_3}$$

$$\Rightarrow V_3 = \left(\frac{a_3 x_3}{a_1 x_1}\right) \times V_1$$

$$V_3 = \left(\frac{-4 \times 4.2}{1.5 \times 3.6}\right) V_1$$

$V_3 = -7/9\,V_1$

Taking moments about P:

$(15 \times 7.8) + (30 \times 4.8) + (33 \times 1.8) + (V_1 \times 0) - (2/9 \times V_1 \times 4.2) - (7/9 V_1 \times 7.8) = 0$

$V_1 = 45.77$ kN

$V_2 = -2/9 V_1 = -10.17$ kN

$V_3 = -7/9 V_1 = -35.59$ kN.

Net axial forces at the based on the building frame:-

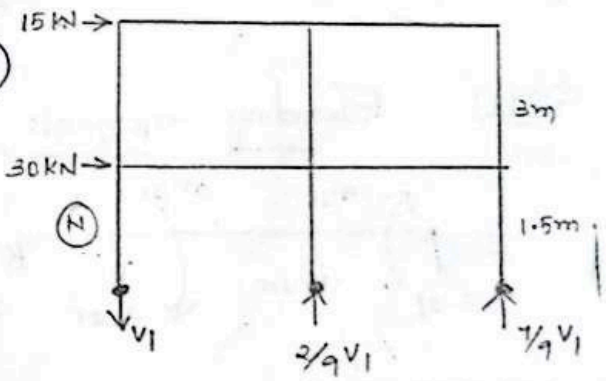
$V_1 + V_2 + V_3$

$= 45.77 - 10.17 - 35.59 = 0$.

Taking Moments about (N)

$(15 \times 4.5) + (30 \times 1.5) + (V_1 \times 0) - (2/9 V_1 \times 4.2)$

$-(7/9 V_1 \times 7.8) = 0$



$V_1 = 16.07$ kN.

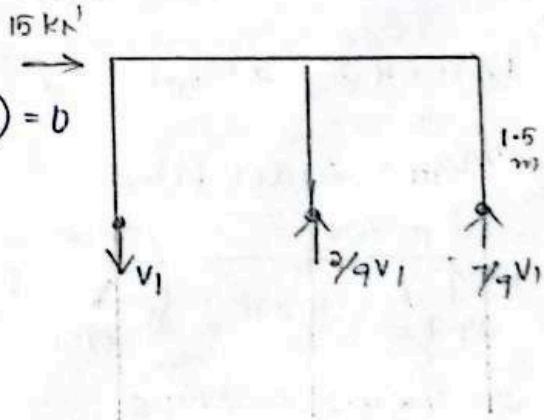
$V_2 = -2/9 V_1 = -3.571$ kN.

$V_3 = -7/9 V_1 = -12.498$ kN.

Net axial forces at the based on the building frome

$16.07 - 3.57 - 12.49 = 0$

Taking moments about (M)



$(15 \times 1.5) + (V_1 \times 0) - (2/9 V_1 \times 4.2) - (7/9 V_1 \times 7.8) = 0$

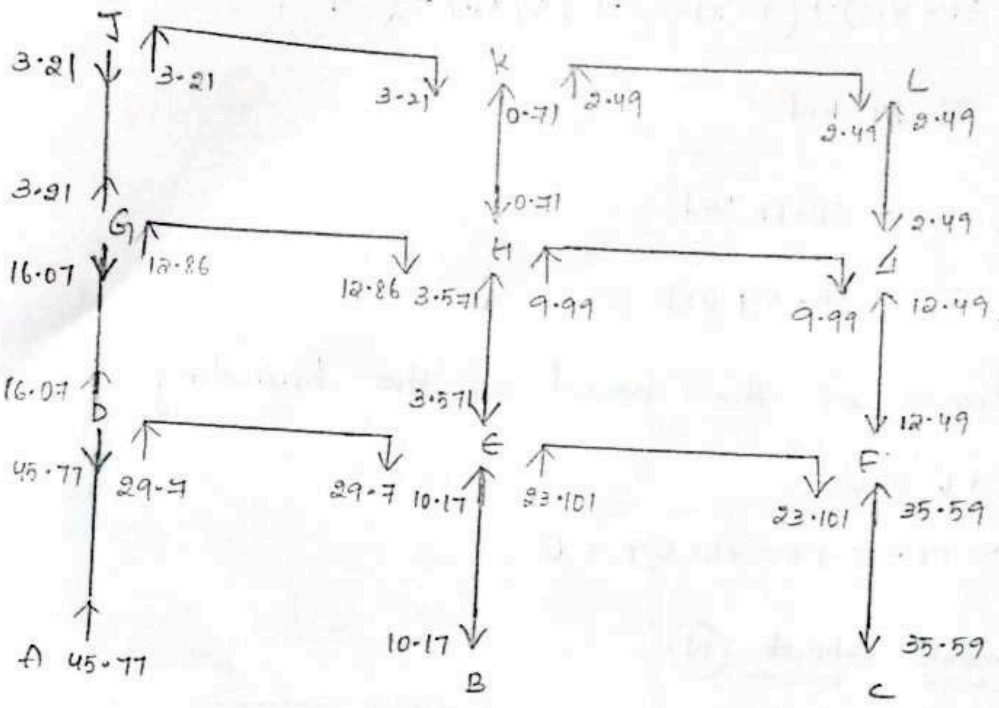
$V_1 = 3.21$ kN

$V_2 = -0.713$ kN

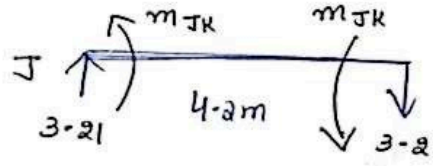
$V_3 = -2.49$ kN. Net axial forces

$= 3.21 - 0.713 - 2.49 = 0$.

Step ④ : Shear forces in beams :-

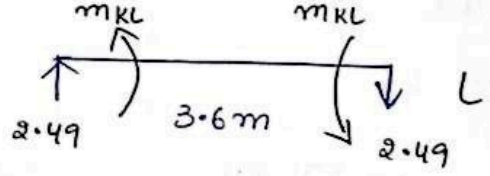


Step ⑤ :- Bending moments in beams (m) :-



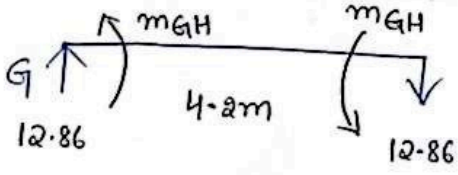
$3.21 \times 4.2 = 2m_{JK}$

$m_{JK} = 6.741$ KN.m



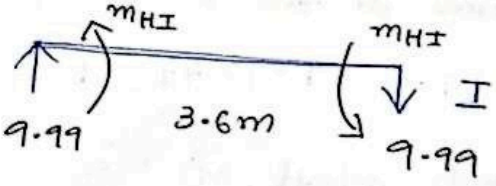
$2.49 \times 3.6 = 2m_{KL}$

$m_{KL} = 4.482$ KN.m



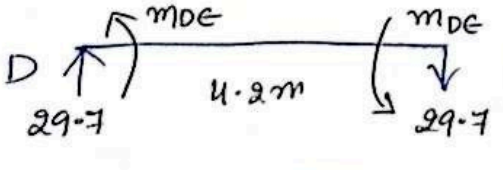
$12.86 \times 4.2 = 2m_{GH}$

$m_{GH} = 27.006$ KN.m



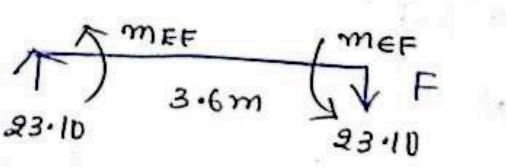
$9.99 \times 3.6 = 2m_{HI}$

$m_{HI} = 17.982$ KN.m.



$29.7 \times 4.2 = 2m_{DE}$

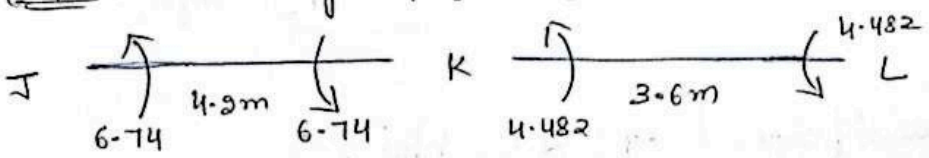
$m_{DE} = 62.37$ KN.m



$23.10 \times 3.6 = 2 \times m_{EF}$

$m_{EF} = 41.58$ KN.m.

Step ⑤: Bending moment in Columns (M).



Joint J:

$M_{JI} + M_{JK} \quad M_{JG_1} = M_{JK}$

$M_{JG_1} = 6\text{-}741 \text{ kN.m}$

Joint K:-

$M_{KH} = M_{KJ} + M_{KL}$

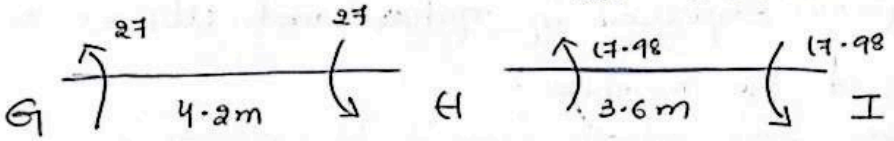
$= 6\text{-}74 + 4\text{-}48$

$M_{KH} = 11\text{-}22 \text{ kN.m}$

Joint L

$M_{LI} = M_{LK}$

$M_{LI} = 4\text{-}482 \text{ kN.m}$



Joint G:-

$M_{GD} + M_{GJ} = M_{GH}$

$M_{GD} = 27 - 6\text{-}741$

$M_{GD} = 20\text{-}259 \text{ kN.m}$

Joint H:

$M_{HE} + M_{HK} = M_{HI} + M_{HG}$

$M_{HE} = 17\text{-}98 + 27 - 11\text{-}22$

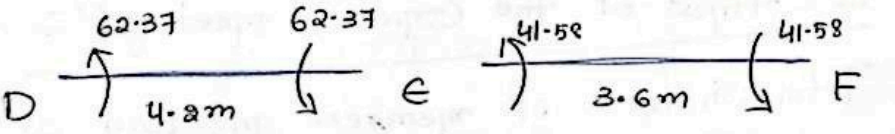
$M_{HE} = 33\text{-}76 \text{ kN.m}$

Joint I

$M_{IE} + M_{IL} = M_{IH}$

$M_{IE} = 17\text{-}98 - 4\text{-}48$

$M_{IE} = 13\text{-}5 \text{ kN.m}$



Joint D:-

$M_{DA} + M_{DG_1} = M_{DE}$

$M_{DA} = 62\text{-}37 - 20\text{-}25$

$M_{DA} = 42\text{-}12 \text{ kN.m}$

Joint E:

$M_{EB} + M_{EH} = M_{EE} + M_{ED}$

$M_{EB} = 41\text{-}58 + 62\text{-}37 - 33\text{-}76$

$M_{EB} = 70\text{-}19 \text{ kN.m}$

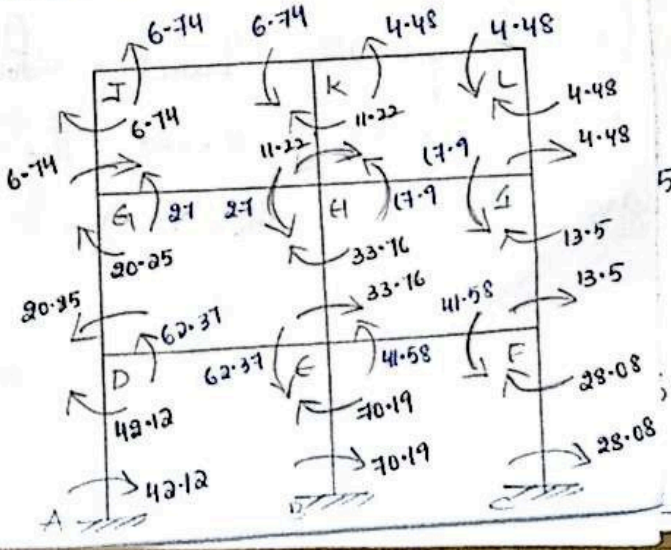
Joint F:

$M_{FC} + M_{FI} = M_{FE}$

$M_{FC} = 41\text{-}58 - 13\text{-}5$

$M_{FC} = 28\text{-}08 \text{ kN.m}$

Step ⑥: BM in Beams & Columns



Factor Method:-

Dt:- 25/11/22

This method was proposed by J.B. Wilbor.

⇒ This approximate method may be used when the member dimensions and the relative stiffness of the members is known.

⇒ This method is also called as 'k' factor method.

⇒ This method is Semi-Empirical in nature and utilizes relative stiffness values of the members.

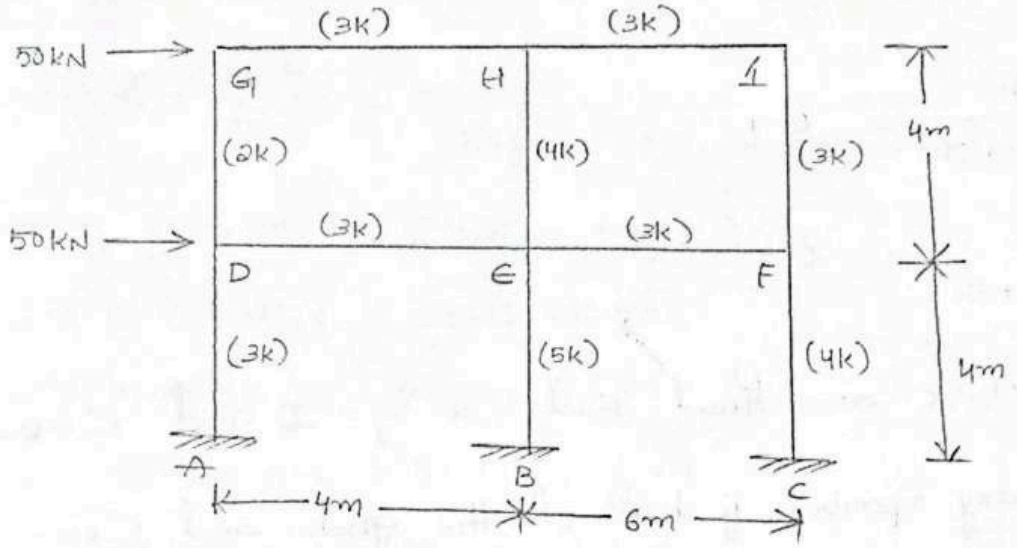
$$\text{Girder factor } (g) = \frac{\Sigma \text{ Sum of the Stiffness of Columns meeting @ Joint}}{\text{Total Stiffness of members meeting at that Joint.}}$$

$$\text{Column factor } (c) = \frac{\Sigma \text{ Stiffness of the Girders meeting @ Joint}}{\text{Total Stiffness of members meeting at Joint}}$$

⇒ At any Joint $\boxed{c+g=1}$

⇒ For fixed Ends $c=1$ and $g=1$

Q:- Analyse the building frame shown in the figure below using factor method. The stiffness of various members are shown in the ( ).



Step ①: Calculation of Column factor and girders factor.

Joint G:- $q = \frac{2k}{3k+2k} = \frac{2k}{5k} = 0.4$

$C = \frac{3k}{2k+3k} = \frac{3k}{5k} = 0.6$

Joint H:- $q = \frac{4k}{3k+3k+4k} = \frac{4k}{10k} = 0.4$

$C = \frac{3k+3k}{3k+3k+4k} = \frac{6k}{10k} = 0.6$

Joint 1:- $q = \frac{3k}{3k+3k} = \frac{3k}{6k} = 0.5$

$C = \frac{3k}{6k} = 0.5$

Joint D:- $q = \frac{3k+2k}{3k+3k+2k} = 0.625$

$C = \frac{3k}{3k+3k+2k} = 0.375$

Joint E:- $q = \frac{5k+4k}{3k+4k+3k+5k} = 0.6$

$C = \frac{3k+3k}{15k} = 0.4$

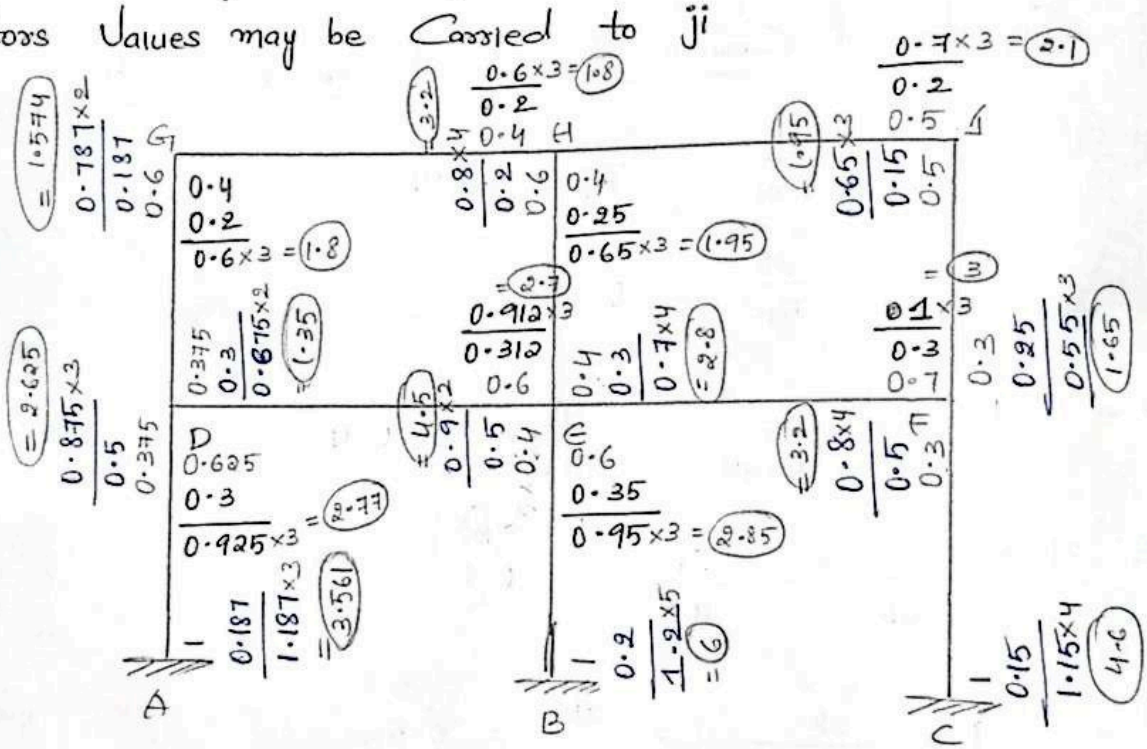
Joint F:-

$$q = \frac{3k+4k}{3k+4k+3k} = 0.7$$

$$c = \frac{3k}{3k+4k+3k} = 0.3$$

⇒ Joint A, B, C are fixed ends so $q = 1$ and $c = 0.1$

Step ②:- For any member ij half of the girder and Column factors values may be carried to ji



Step ③:-

Column - Moment factor (C_m) = Step ② × relative stiffness

Girder - Moment factor (g_m) = Step ② × relative stiffness.

Step ④:- BM's in the Columns.

B.M in any Column $M_{ij} = A \cdot C_m$.

Where $A = \frac{S_s \cdot h}{\Sigma C_m}$

S_s = Storey shear

h = height of the storey.

Upper Storey:-

$$\text{Constant } A = \frac{S_s \cdot h}{\Sigma C_m} = \frac{50 \times 4}{1.576 + 1.35 + 3.2 + 2.8 + 1.95 + 1.65} = 15.96$$

Bending moment in Column $M_{ij} = A \cdot C_m$.

$M_{GD} = 15.96 \times 1.576 = 25.16$ kN·m.

$M_{DG} = 15.96 \times 1.35 = 21.546$ kN·m

$M_{HE} = 15.96 \times 3.2 = 51.072$ kN·m.

$M_{EH} = 15.96 \times 2.8 = 44.688$ kN·m.

$M_{IF} = 15.96 \times 1.95 = 31.122$ kN·m

$M_{FI} = 15.96 \times 1.65 = 26.334$ kN·m.

Lower Storey:-

$$A = \frac{(50+50) \times 4}{2.625 + 3.567 + 4.5 + 6 + 3.2 + 4.6} = 16.33$$

$M_{DA} = 16.33 \times 2.625 = 42.86$ kN·m.

$M_{AD} = 16.33 \times 3.567 = 58.25$ kN·m.

$M_{EB} = 16.33 \times 4.5 = 73.48$ kN·m

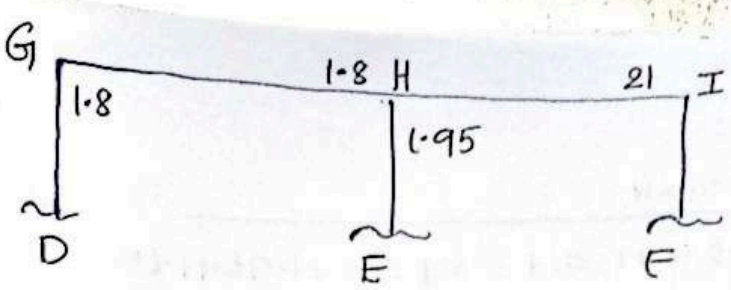
$M_{BE} = 16.33 \times 6 = 97.98$ kN·m

$M_{FC} = 16.33 \times 3.2 = 52.26$ kN·m

$M_{CF} = 16.33 \times 4.6 = 75.12$ kN·m.

Step ⑤: BM's in beams "m_{ij}"

Distribute the moment in Proportion to their girder-moment values.



$$m_{HI} = \frac{1.95}{1.8+1.95}(51.07)$$

$$= 26.56 \text{ kN·m}$$

$M_{GD} = m_{GH}$

$m_{GH} = 25.16$ kN·m

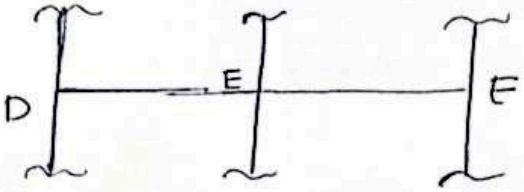
$$m_{HG} = \frac{1.8}{1.8+1.95} M_{HE}$$

$$= \frac{1.8 \times 51.07}{1.8+1.95}$$

$$= 24.51 \text{ kN·m}.$$

$m_{IH} = M_{IF}$

$m_{IH} = 31.22$.



$m_{DE} = M_{DG} + M_{DA}$

$= 21.546 + 42.86$

$= 64.4$ kN·m.

$m_{FE} = M_{FI} + M_{FC}$

$= 26.32 + 52.26$

$= 78.59$ kN·m

$$M_{ED} = \frac{2.739}{2.739+2.85}(M_{EB} + M_{EH})$$

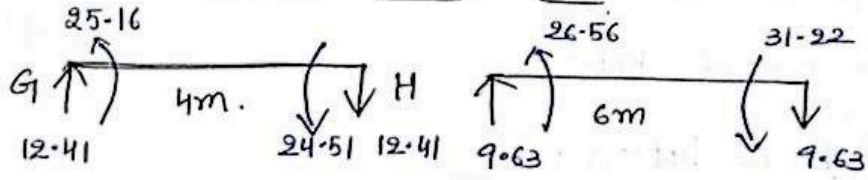
$$= 0.49(73.48 + 44.68)$$

$$= 57.9 \text{ kN·m}.$$

$$M_{EE} = \frac{2.85}{2.739+2.85}(73.48 + 44.68)$$

$$= 60.25 \text{ kN·m}.$$

Step(6): Shear forces in beams:

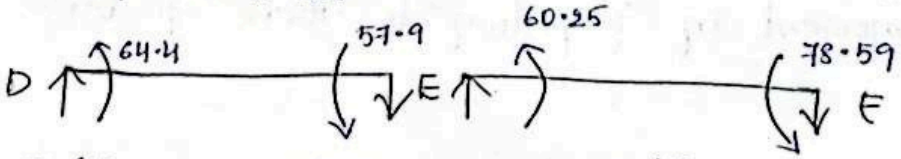


$R_1(4) = 25.16 + 24.51$

$R_1 = 12.41$ kN.

$R_1(6) = 26.56 + 31.22$

$= 9.63$ kN.



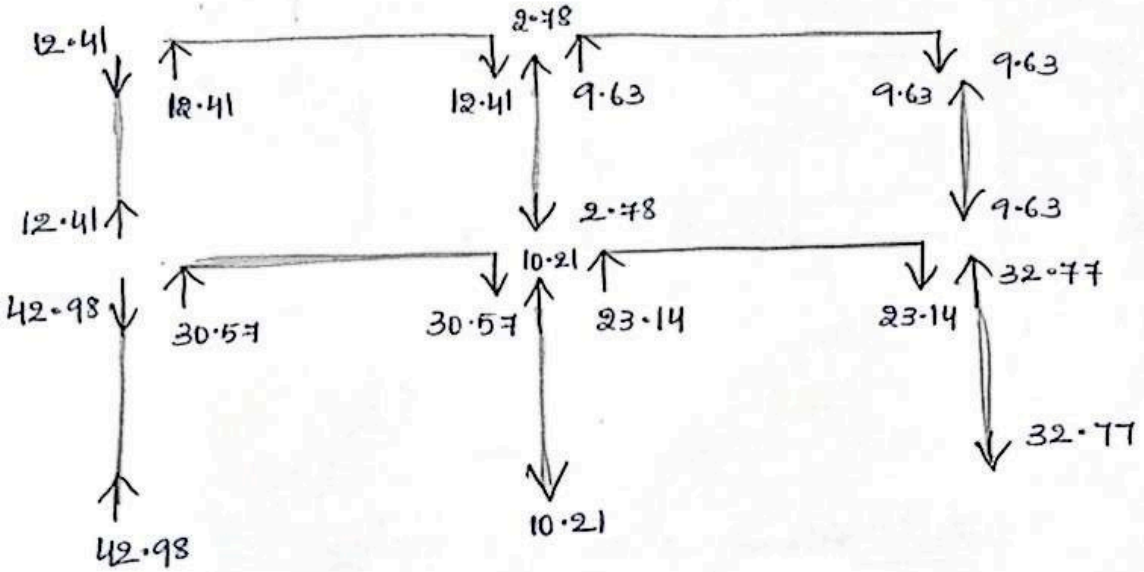
$R_2(4) = 64.4 + 57.9$

$R_2 = 30.57$ kN

$R_2(6) = 60.25 + 78.59$

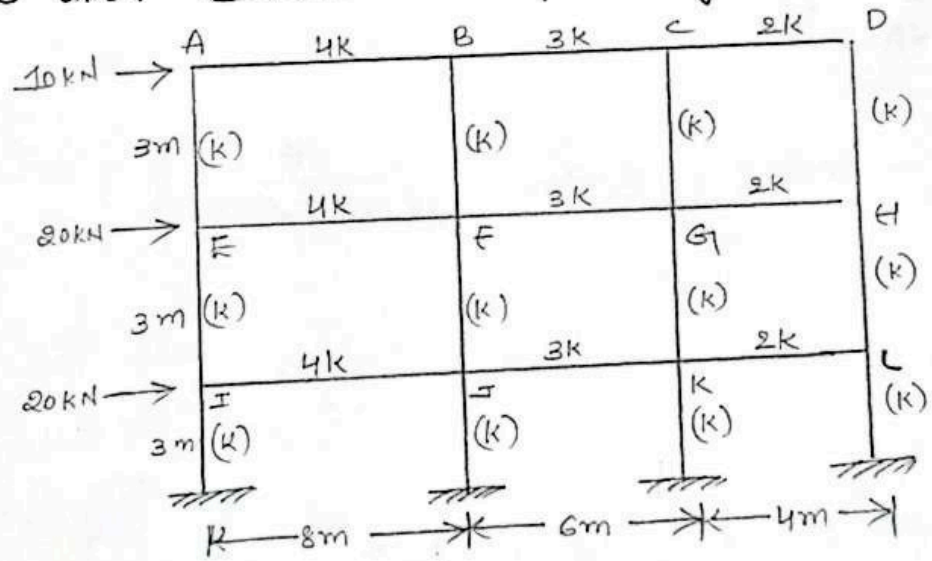
$R_2 = 23.14$ kN.

Step (7): Axial forces in Columns :-

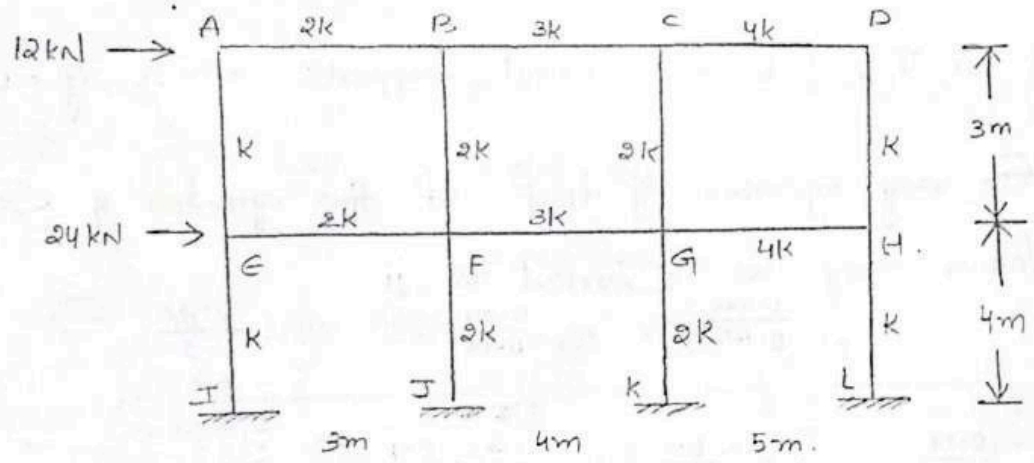


Net axial forces at base = 42.98 − 10.21 − 32.77 = 0.

Q: Analyze the frame by factor method to get moments in girders and Caumns of top Storey.



Q:- Analyze the frame shown in figure by factor method. Q1:- 30/11/2022



Sol:- Step ① :- Findout Columns & Girder factors:-

Joint A:- $q = \frac{K}{K+2K} = 0.33$ $\quad C = \frac{2K}{K+2K} = 0.67$

Joint B:- $q = \frac{2K}{2K+3K+2K} = 0.28$ $\quad C = \frac{2K+3K}{2K+2K+3K} = 0.71$

Joint C:- $q = \frac{2K}{3K+4K+2K} = 0.2$ $\quad C = \frac{3K+4K}{3K+4K+2K} = 0.7$

Joint D:- $q = \frac{K}{4K+K} = 0.2$ $\quad C = \frac{4K}{K+4K} = 0.8$

Joint E:- $q = \frac{K+K}{2K+K+K} = 0.5$ $\quad C = \frac{2K}{K+K+2K} = 0.5$

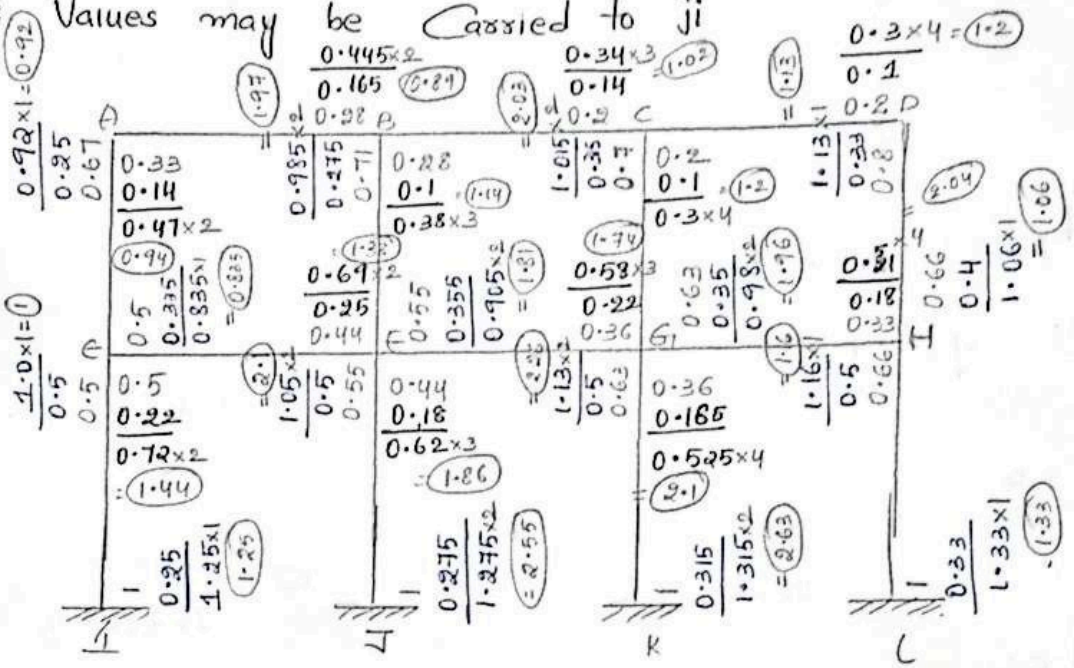
Joint F:- $q = \frac{2K+2K}{2K+3K+2K+2K} = 0.44$ $\quad C = \frac{2K+3K}{2K+2K+3K+2K} = 0.55$

Joint G:- $q = \frac{2K+K2K}{3K+4K+2K+2K} = 0.36$ $\quad C = \frac{3K+4K}{11K} = 0.63$

Joint E:- $q = \frac{K+K}{4K+2K} = 0.33$ $\quad C = \frac{4K}{4K+K+K} = 0.66$

⇒ Joint I, J, K & L are fixed Supports. $C = 1, q = 0$

Step ②:- For any member ij half of the girders & Column factors Values may be Carried to ji



Step ③:-

Column - Moment factor = (c_m) = Step ② x relative Stiffness.

Girder - Moment factor (g_m) = Step ② x relative Stiffness.

Step ④: BM's in Column's (M_{ij})

$$\text{Constant } A = \frac{Sh}{\Sigma C_m} =$$

Upper Storey:- $$A = \frac{12 \times 3}{0.92 + 0.835 + 1.97 + 1.81 + 2.03 + 1.96 + 1.13 + 1.06}$$

$$A = 3.04$$

$M_{AE} = 0.92 \times 3.04 = 2.79$ kN·m

$M_{EA} = 0.835 \times 3.04 = 2.53$ kN·m

$M_{BF} = 1.97 \times 3.04 = 5.98$ kN·m

$M_{FB} = 1.81 \times 3.04 = 5.50$ kN·m

$M_{CG} = 2.03 \times 3.04 = 6.17$ kN·m

$M_{GC} = 1.96 \times 3.04 = 5.95$ kN·m

$M_{DH} = 1.13 \times 3.04 = 3.43$ kN·m

$M_{HD} = 1.06 \times 3.04 = 3.22$ kN·m

Lower Storey:- $A = \dfrac{(12+24) \times 4}{1+1.25+2.1+2.55+2.26+2.63+1.16+1.33}$

$A = 10.08$

$M_{EI} = 10.08 \times 1 = 10.08$ kN·m

$M_{IE} = 10.08 \times 1.25 = 12.6$ kN·m

$M_{FJ} = 10.08 \times 2.1 = 21.168$ kN·m

$M_{JF} = 10.08 \times 2.55 = 25.704$ kN·m

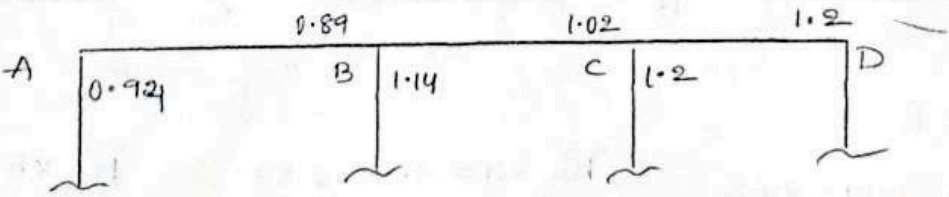
$M_{GK} = 10.08 \times 2.26 = 22.78$ kN·m

$M_{KG} = 10.08 \times 2.63 = 26.51$ kN·m

$M_{HL} = 10.08 \times 1.16 = 11.69$ kN·m

$M_{LH} = 10.08 \times 1.33 = 13.40$ kN·m

Step ⑤:- BM's in Beams (m)



$m_{AB} = M_{AE} = 2.79$ kN·m

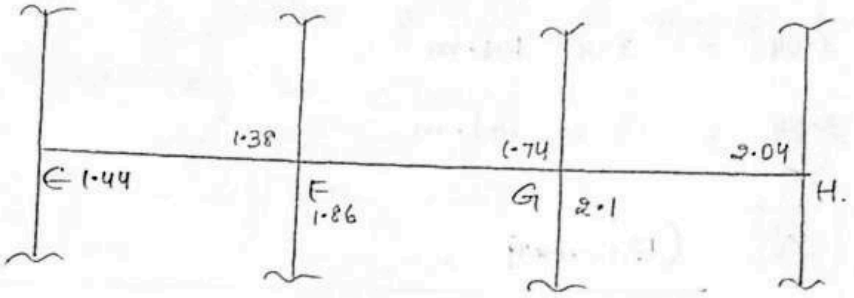
$m_{BA} = \dfrac{0.89}{0.89+1.14} M_{BF} = 2.62$ kN·m

$m_{BC} = \dfrac{1.14}{0.89+1.14} (M_{BF}) = 3.35$ kN·m

$$m_{CB} = \frac{1.02}{1.02+1.2} M_{CG_1} = 2.83 \text{ kN.m}$$

$$m_{CD} = \frac{1.2}{1.02+1.2} M_{CG_1} = 3.33 \text{ kN.m}$$

$$m_{DC} = M_{DH} = 3.43 \text{ kN.m.}$$



$$m_{EE} = M_{EA} + M_{EI}$$

$$m_{EE} = 2.53 + 10.08$$

$$= 12.61 \text{ kN.m.}$$

$$m_{FE} = \frac{1.38}{1.38+1.86} \times (M_{FB} + M_{FJ})$$

$$m_{FE} = 11.35 \text{ kN.m.}$$

$$m_{FG} = \frac{1.86}{1.38+1.86} \times (M_{FB} + M_{FJ})$$

$$= 15.30 \text{ kN.m.}$$

$$m_{GF} = \frac{1.74}{1.74+2.1} \times (M_{GC} + M_{GK})$$

$$= 13.01 \text{ kN.m.}$$

$$m_{GH} = \frac{2.1}{1.74+2.1} \times (M_{GH} + M_{GK})$$

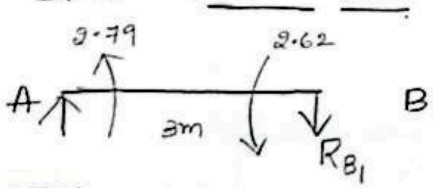
$$= 15.71 \text{ kN.m.}$$

$$m_{HG} = M_{HD} + M_{HL}$$

$$= 3.22 + 11.69$$

$$= 14.91 \text{ kN.m.}$$

Step ⑥ :- Shear force

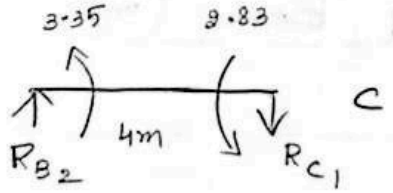


$$\Sigma M_B = 0$$

$$R_A \times 3 = 2.79 + 2.62$$

$$R_A = 1.80 \text{ kN}$$

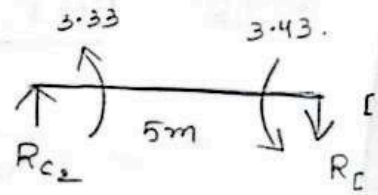
$$R_{B_1} = -1.80 \text{ kN}$$



$$R_{B_2} \times 4 = 3.35 + 2.83$$

$$R_{B_2} = 1.545 \text{ kN}$$

$$R_{C_1} = -1.54 \text{ kN}$$

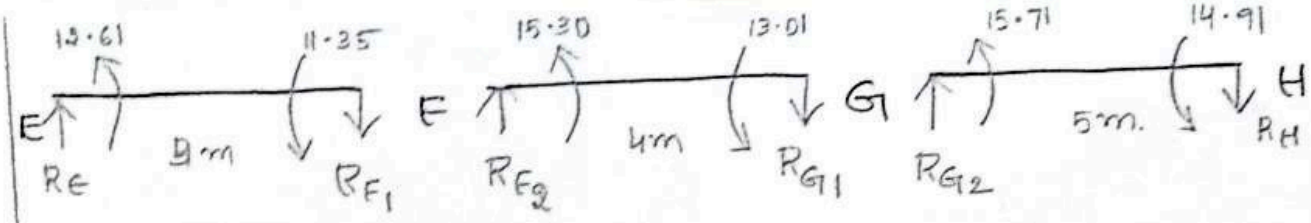


$$R_{C_2} \times 5 = 3.33 + 3.4$$

$$R_{C_2} = 1.35 \text{ kN}$$

$$R_D = -1.35 \text{ kN}$$

$M_{EA} = 0.835 \times 3.04 = 2.53$ kN.m



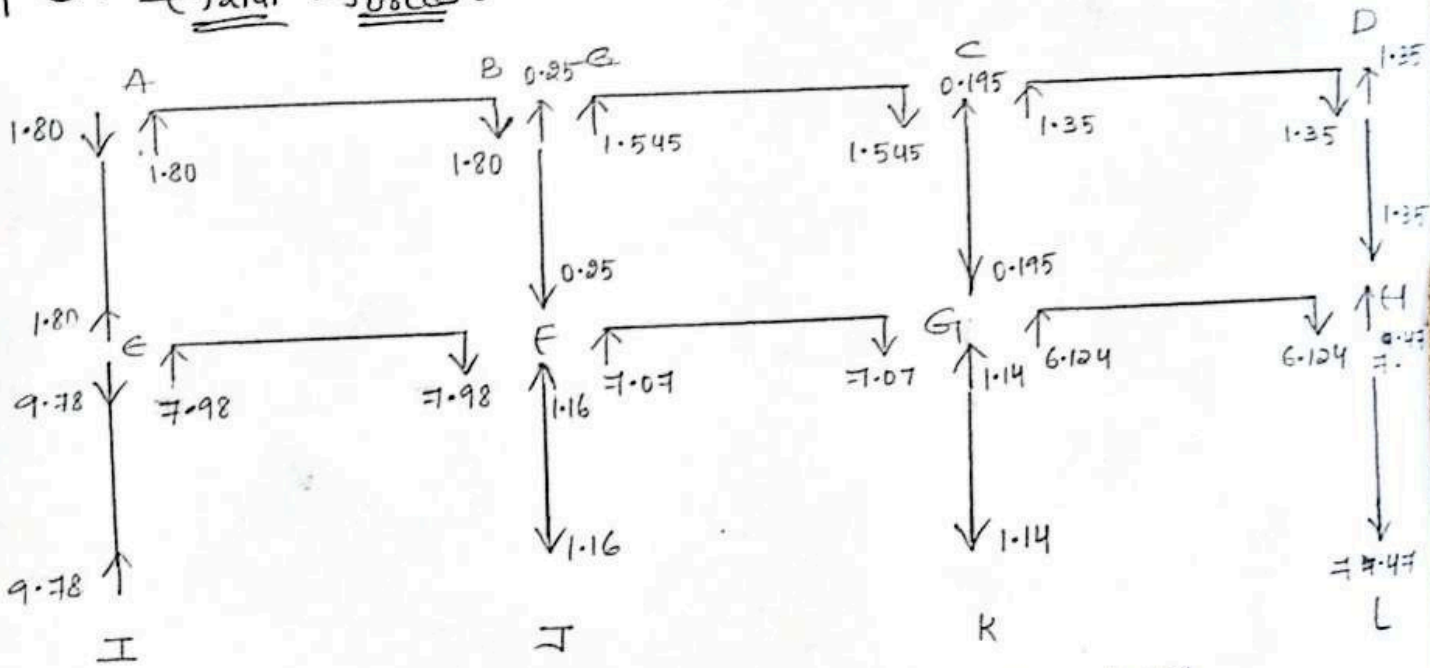
$\Sigma M_E = 0$ CW = ACW $R_{G_2} \times 5 = 15.71 + 14.91$

$R_E \times 3 = 12.61 + 11.35$ $R_{F_2} \times 4 = 15.3 + 13.01$ $R_{G_2} = 6.124$ kN

$R_E = 7.98$ kN $R_{F_2} = 7.07$ kN $R_H = -6.124$ kN

$R_{F_1} = -7.98$ kN $R_{G_1} = -7.07$ kN

Step ④ :- Axial forces :-

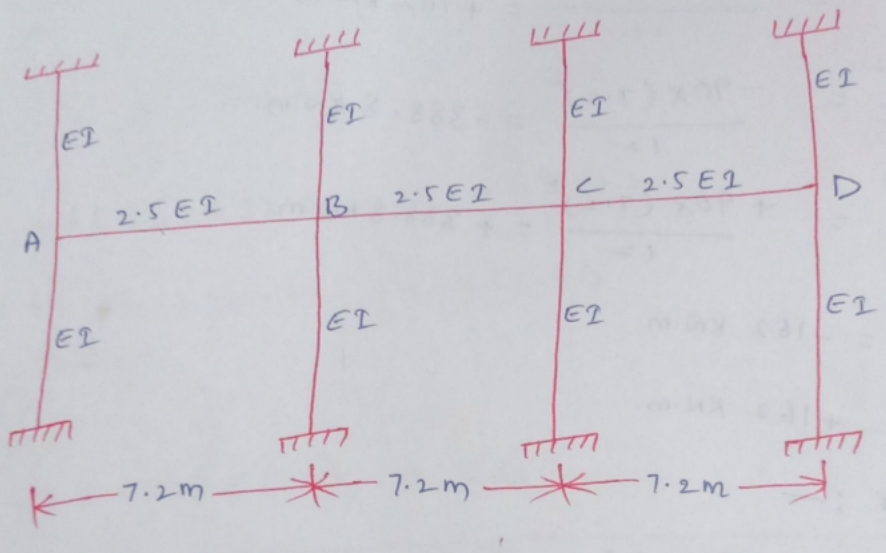


Net axial forces at base = 9·78 − 1·16 − 1·14 − 7·47

= 0.

Substitute frame Method :—

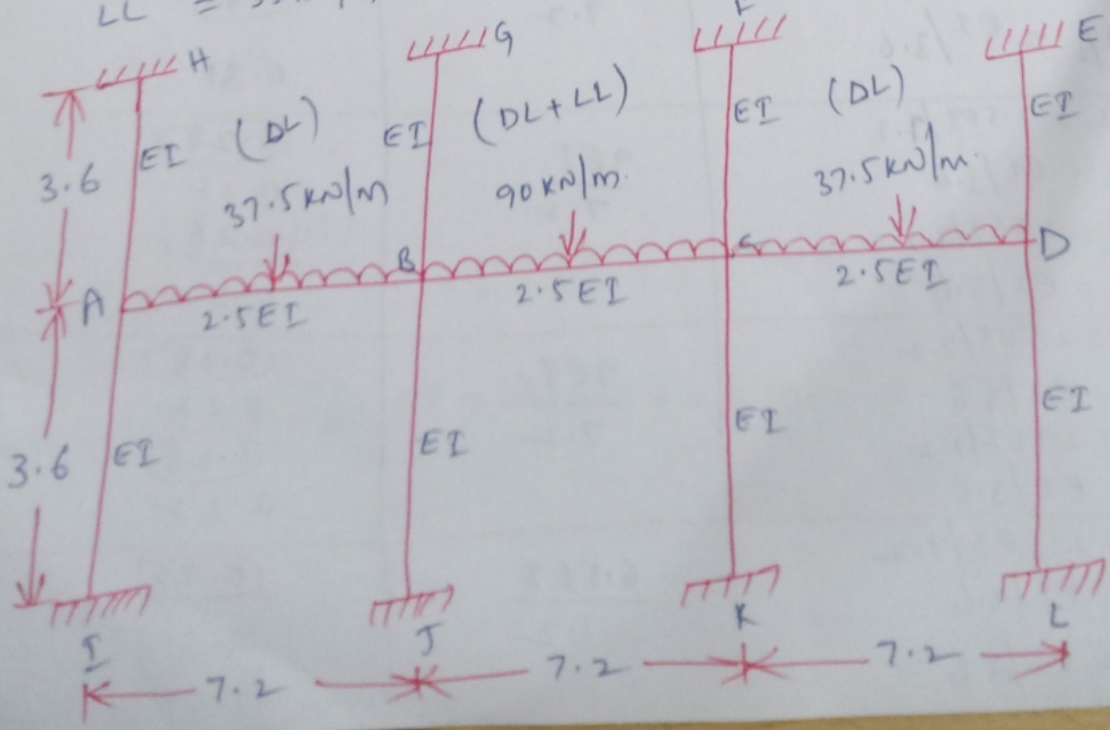
(1) Determine the design sagging moment for the central span of the continuous beam shown by substitute frame method. The beams carry a dead load (including self weight) of 25 KN/m and live load of 35 KN/m. EI of beam is 2.5 times of columns and length of column is 3.6 m.



(A) Factored load

DL = 25 KN/m = 25 × 1.5 = 37.5 KN/m

LL = 35 KN/m = 35 × 1.5 = 52.5 KN/m.



Fixed end moments :—

Columns :

$$M_{f_{AH}} = M_{f_{AI}} = M_{f_{BG}} = M_{f_{BJ}} = M_{f_{CF}} = M_{f_{CK}} = M_{f_{DE}} = M_{f_{DL}} = 0$$

Beams :

$$M_{f_{AB}} = \frac{-wl^2}{12} = \frac{-37.5 \times (7.2)^2}{12} = -162 \text{ KN.m}$$

$$M_{f_{BA}} = \frac{+wl^2}{12} = \frac{+37.5 \times (7.2)^2}{12} = +162 \text{ KN.m}$$

$$M_{f_{BC}} = \frac{-wl^2}{12} = \frac{-90 \times (7.2)^2}{12} = -388.8 \text{ KN.m}$$

$$M_{f_{CB}} = \frac{+wl^2}{12} = \frac{+90 \times (7.2)^2}{12} = +388.8 \text{ KN.m}$$

$$M_{f_{CD}} = \frac{-wl^2}{12} = -162 \text{ KN.m}$$

$$M_{f_{DC}} = \frac{+wl^2}{12} = +162 \text{ KN.m}$$

Rotation factor :—

Joint	Member	R.S.(K)	T.S.(ΣK)	R.F. $= \frac{K}{\Sigma K}$
A	AH	$I/L = EI/3.6$	$\frac{6.5EI}{7.2}$	0.31
	AB	$I/L = 2.5EI/7.2$		0.38
	AI	$I/L = EI/3.6$		0.31
B	BA	$I/L = 2.5EI/7.2$	$\frac{9EI}{7.2}$	0.28
	BG	$I/L = EI/3.6$		0.22
	BC	$I/L = 2.5EI/7.2$		0.28
	BJ	$I/L = EI/3.6$		0.22
C	CB	$I/L = 2.5EI/7.2$	$\frac{9EI}{7.2}$	0.28
	CF	$I/L = EI/3.6$		0.22
	CD	$I/L = 2.5EI/7.2$		0.2
	CK	$I/L = EI/3.6$		0.2
D	DC	$I/L = 2.5EI/7.2$	$\frac{6.5EI}{7.2}$	0.28
	DE	$I/L = EI/3.6$		31
	DL	$I/L = EI/3.6$		.31

Iteration Process :—

A	B		C		D
0.38	0.28	0.28	0.28	0.28	0.38
	+162	-388.8	+388.8	-162	+162
-162					
+61.56	+64.04	+64.04	-66.04	-64.04	-61.56
+32.02	+30.78	-32.02	+32.02	-30.78	-32.02
-68.42	256.84	-356.78	356.78	-256.82	68.42
26	28	+28	-28	-28	-26
14	13	-14	+14	-13	-14
-28.42	297	-343	+343	-297	-28

Simply supported moments :—

$$M_{AB} = \frac{wl^2}{8} = \frac{37.5 \times (7.2)^2}{8} = 243$$

$$M_{BC} = \frac{wL^2}{8} = \frac{90 \times (7.2)^2}{8} = 583.2$$

$$M_{CD} = \frac{wl^2}{8} = \frac{37.5 \times (7.2)^2}{8} = 243$$

Net sagging moment for BC

$583.2 - 343 = 240$ KN.M.

calculations :—

cycle (1) :-

$-162 \times 0.38 \times (-1) = +61.56$

$+162 + 8(-388.8) = -550.8$

388.8
162.0
+
550.8

$-550.8 \times 0.28 \times (-1) = +64.04$

$\frac{64.04}{2} = +32.02$

$\frac{61.56}{2} = 30.78$

$-162 + 61.56 + 32.02 = -68.42$

$+162 + 64.04 + 30.78 = 256.84$

356.78
256.84
99.94

cycle (2) :-

$-68.42 \times 0.38 \times (-1) = 26$

$256.84 - 356.78 = -99.94$

$-99.94 \times 0.28 \times (-1) = 28$

$\frac{28}{2} = 14$

$\frac{26}{2} = 13$

Final moments :-

$-162 + 61.56 + 32.02 - 68.42 + 26 + 14 = -28.42$

$+162 + 64.04 + 30.78 + 256.84 + 28 + 13 = 297$

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