TEEGALA KRISHNA REDDY ENGINEERING COLLEGE

(UGC-Autonomous)

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DEPARTMENT OF CIVIL ENGINEERING
LECTURE NOTES

ACADEMIC YEAR: 2023-24

REGULATION: R20

YEAR/SEMESTER: III - I

SUBJECT: **STRUCTURALANALYSIS– II**

SUBJECT CODE: **20CE5PC04**

FACULTY NAME: P.CHAITANYA

SYLLABUS:

UNIT – IV

MatrixMethodsOfAnalysis: IntroductiontoFlexibilityandStiffnessmatrix methodsofanalysesusing 'system approach' up to three degree of indeterminacy– Analysis of continuous beams includingsettlementofsupportsusingflexibilityandstiffnessmethods– Analysis of pin-jointed determinate plane frames using flexibility and stiffness methods- Analysis of single bay single storey portal frames using stiffness method–Shear force and bending moment diagrams – Elastic curve.

Unit-04

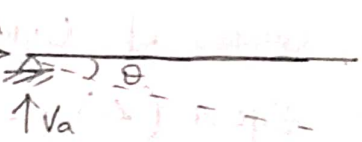
:- Matrix Method of Analysis :-

Types of Supports :-

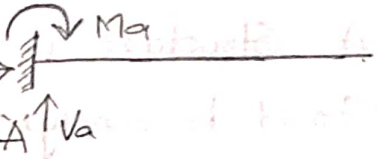
1] Roller Support



2] Hinged Support (or) Pinned Support



3] Fixed Support (or) rigid Support
(or) Built in Support



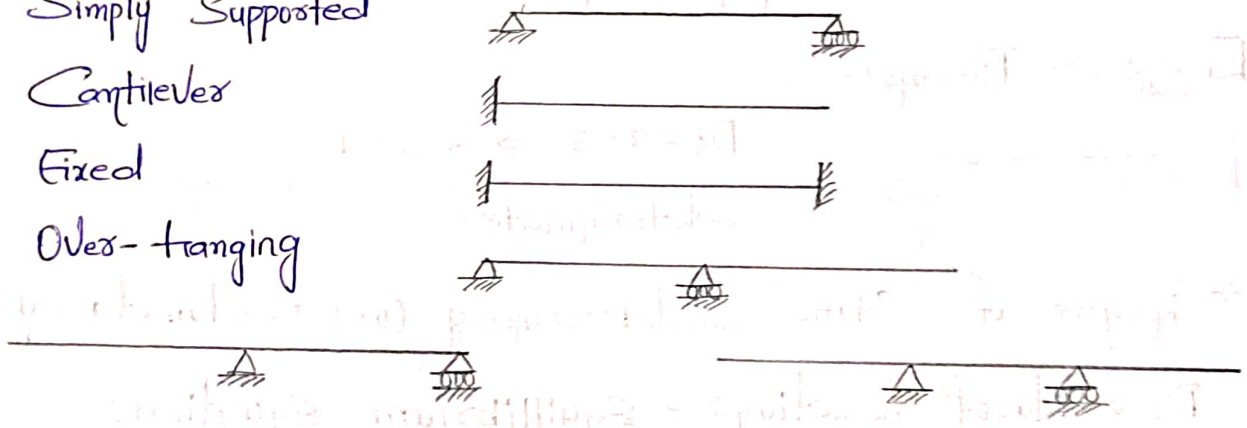
Types of Beams :-

1] Simply Supported

2) Cantilever

3) Fixed

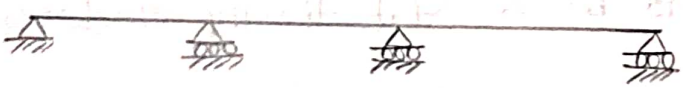
4) Over-hanging



5] Continuous Beam :-



→ Two Span Continuous beam



→ 3 Span Continuous beam.

6] Propped Cantilever beam



Equillibrium Equations :-

$\Sigma f_x = 0$ (or) $\Sigma H = 0$ (or) $\leftarrow = \rightarrow$

$\Sigma f_y = 0$ (or) $\Sigma V = 0$ (or) $\uparrow = \downarrow$

$\Sigma M_a = 0$ (or) $\circlearrowright = \circlearrowleft$

Determinate (or) Indeterminate Structure:-

→ A Structure is Said to be a determinate Structure, If it Can be analyzed with the help of Equillibrium Equations alone (only).

As the Equillibrium Equations are Only 3 as listed above the number of Unknows that Can be found Out are less than (or) Equal ($\leq$) 3.

→ A Structure is Said to be a indeterminate Structure. If It Cannot be analyzed with the help of Equillibrium Equations "As the No: of Unknows in Such Structure are > 3."

Examples :- Beams:-

1] 

$D_s = r - 3 \Rightarrow 3 - 3 = 0$

determinate.

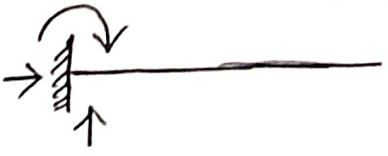
→ Degree of Static Indeterminacy (or) redundancy 'D_s'

D_s = No: of reactions − Equillibrium Equations

$D_s = r - 3$.

→ If $D_s = 0$ it is Said to be a Statistically determinate.

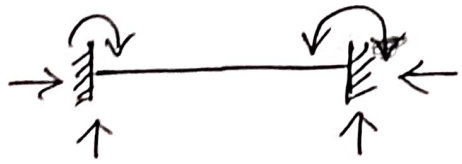
If $D_s \neq 0$ it is Said to be a Statistically Indeterminate.

2] 

$D_s = r - 3$

$= 3 - 3 = 0$

$D_s = 0$, So it is Said to be a Statistically determinate.

3) 

$D_s = r - 3$

$= 6 - 3 = 3$

So, fixed beam is Statistically Indeterminate (or) redundant by 3rd degree.

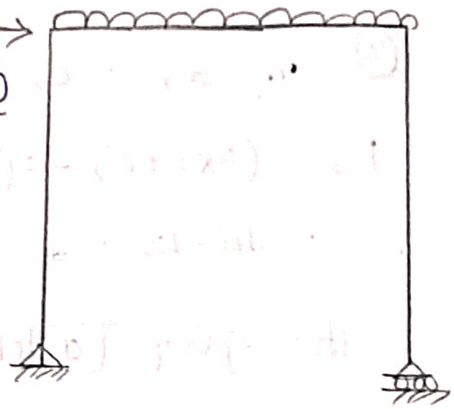
4)  Propped Cantilever beam

$D_s = r - 3 \Rightarrow 4 - 3 = 1$

Statistically Indeterminate by '1' degree.

PORTAL FRAMES [Rigid Jointed] → always 90°.

In - Portal frame Each member of rigid jointed Portal frame has 3 types of internal forces namely axial force, Shear force and bending moment.



Let, m = No: of members

r = No: of reactions

j = No: of Joints.

→ If 'm' members → $(3m+r)$ [Unknown].

Total Unknowns $(3m+r)$ is Solved by Using Equillibrium Equations.

For j Joints = $3j$ (known)

$D_s = (3m+r) - 3j \rightarrow \Sigma f_x = 0, \Sigma f_y = 0, \Sigma z = 0$

A.f, S.f, B.m

→ The total No: of Unknowns = Member forces + reactions

$= 3m+r$

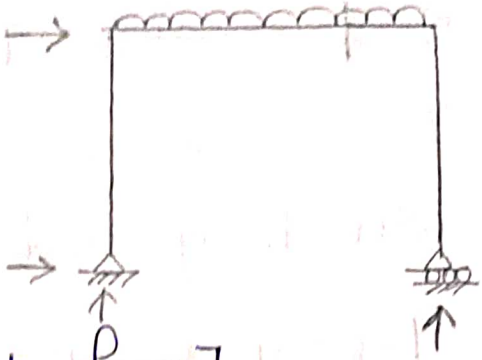
→ Total No: of Equillibrium Equations available $= 3j$

$\therefore D_s = (3m+r) - 3j$

If $D_s = 0$, it is a Statistically determinate frame.

$D_s \neq 0$ it is a Statistically Indeterminate frame.

Examples :-

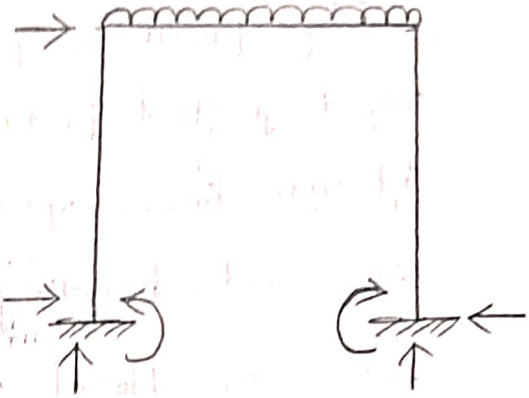


① $m = 3$, $r = 3$, $j = 4$

$D_s = (3m + r) - 3j$

$= (3 \times 3 + 3) - 3(4)$

$D_s = 12 - 12 \Rightarrow 0$ [Statically determinate frame].



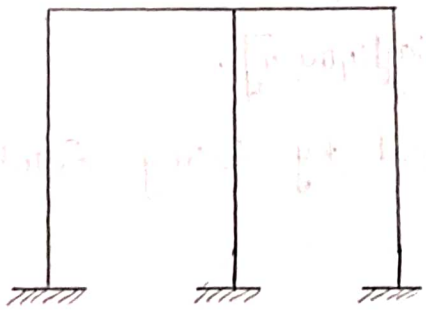
② :- $m = 3$, $r = 6$, $j = 4$

$D_s = (3 \times 3 + 6) - 3(4)$

$= 15 - 12 = 3$

The given portal frame is statistically indeterminate by 3rd degree.

Portal frames [Indeterminate]

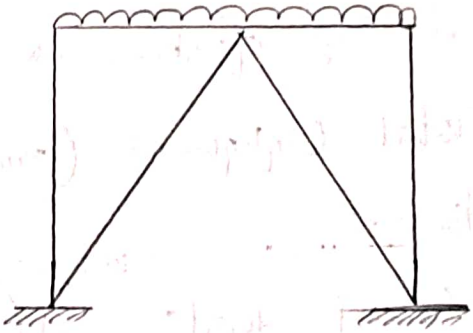


$m = 5$, $r = 9$, $j = 6$

$D_s = (3 \times 5 + 9) - 3(6)$

$D_s = 6$

Statically Indeterminate.



$m = 5$, $r = 6$, $j = 5$

$D_s = [(3)5 + 6] - 3(5)$

$D_s = 6$.

Statically Indeterminate

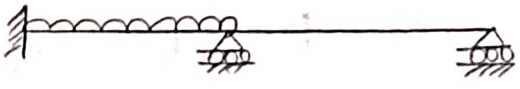
Continuous Beams :-



$D_s = r - 3$

$D_s = 4 - 3 = 1$

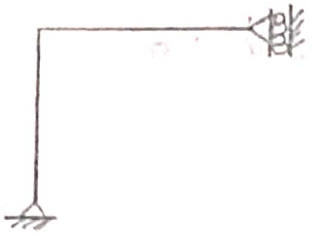
Indeterminate.



$D_s = 5 - 3 = 2$

Indeterminate.

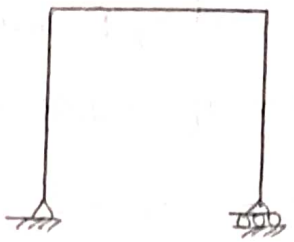
Portal frames [Determinate]



$D_s = [3(2)+3] - 3(3)$

$= 0$

Statically determinate



$D_s = [3(3)+3] - 3(4)$

$= 0$

Statically determinate.

Continuous beams [Determinate]



$D_s = r - 3 \Rightarrow 3 - 3$

$= 0$.

TRUSSES :- [PIN Jointed]

In a Pin jointed frame (or) a truss, members have only axial forces [tension / compression] and loads are assumed to act only at the joints.

Let m, r, j be the No: of members, reactions, & joints in a Pin-Jointed truss, to connect the 1st three joints of a truss we require 3 members, to connect each additional joint after 3, we require only 2 members per joint.

∴ Total No: of members in a truss

$$m = 3 + 2(j-3)$$

$$= 2j-3$$

If $m = 2j-3 \rightarrow$ Stable & Statically determinate internally.

$m < 2j-3 \rightarrow$ Unstable internally

$m > 2j-3 \rightarrow$ Overstiff and Statically Indeterminate internally.

$$(D_s)_i = m - (2j-3)$$

The external reactions are to be found out using the three Equillibrium Equations.

$$(D_s)_e = r - 3$$

Total degree of Static Indeterminacy

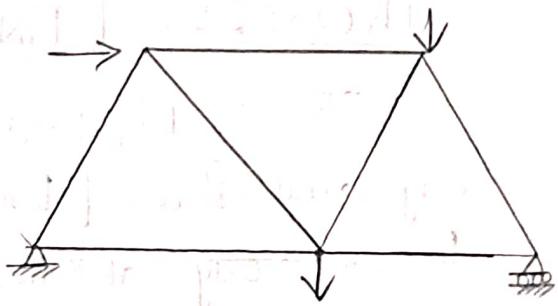
$$D_s = (D_s)_i + (D_s)_e$$

$$D_s = m - (2j-3) + r - 3$$

$$D_s = m + r - 2j$$

Examples :-

① $m = 7,\ r = 3,\ j = 5$



$$(D_s)_i = m - (2j-3)$$
$$= 7 - [(2)(5) - 3]$$
$$= 0$$

Truss is determinate & Stable Internally

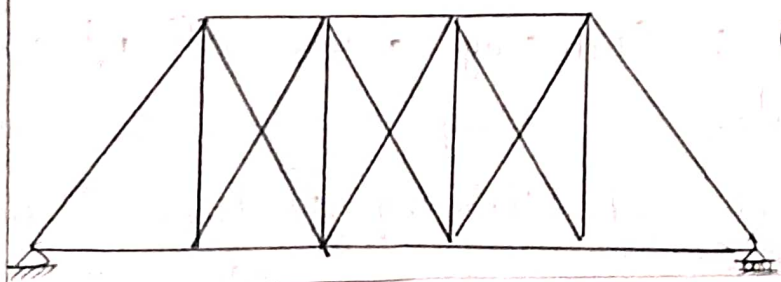
$$(D_s)_e = r - 3$$
$$\Rightarrow 3 - 3 = 0$$

Truss is determinate Externally.

$$D_s = (D_s)_i + (D_s)_e = 0$$

Determinate.

② (Id, d)



$m = 20,\ r = 3,\ j = 10$

$$(D_s)_i = m - (2j-3)$$
$$= 20 - [2(10) - 3] = 3$$

Truss is Indet internally.

$(Ds)_e = r - 3 \Rightarrow 3 - 3 = 0$

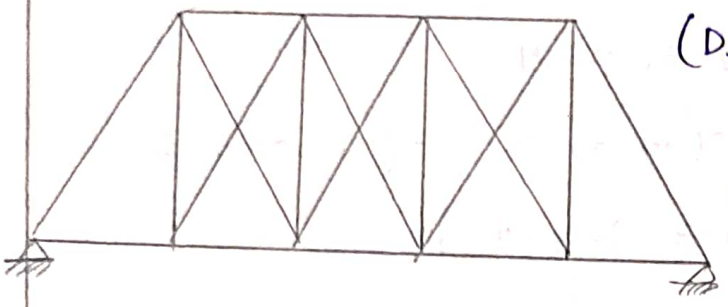
Indeterminate Externally

$Ds = (Ds)_i + (Ds)_e$

$= 3 + 0 = 3$ — Statically Indeterminate.

Eg ③ :- (Id, Id).

$m = 20,\ j = 10,\ r = 4.$



$(Ds)_i = m - (2j - 3) \Rightarrow 20 - [2 \times 10 - 3]$

$= 3$

Statically Indeterminate Internally

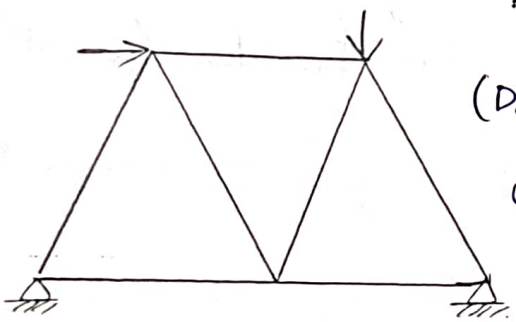
$(Ds)_e = r - 3 \Rightarrow 4 - 3 = 1$

Statically index Externally

$Ds = Dsi + Dse = 3 + 1 = 4.$

④ [d, Id]

$m = 7,\ j = 5,\ r = 4$



$(Ds)_i = m - [2j - 3] = 7 - [2(5) - 3] = 0$

Statically determinate internally.

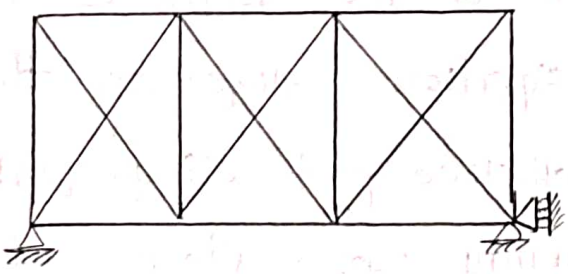
$(Ds)_e = r - 3 \Rightarrow 4 - 3 = 1$

Statically Indeterminate Externally.

$Ds = Dsi + Dse = 0 + 1 = 1$ "

⑤ (Id, d)

$m = 16,\ r = 3,\ j = 8$



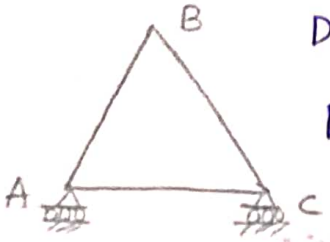
$Dsi = m - [2j - 3] \Rightarrow 16 - [2(8) - 3]$

$= 3$ (Indeterminate)

$Dse = r - 3 \Rightarrow 3 - 3 = 0$

Statically determinate Externally.

⑥ (d, d)



$m = 3,\ j = 3,\ r = 3$

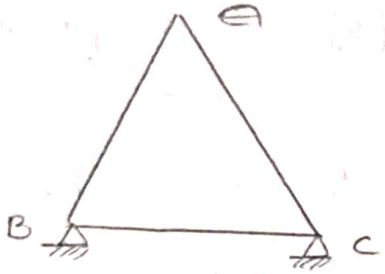
$D_{si} = m - (2j-3) \Rightarrow 3 - [2(3)-3] = 0$

$D_{se} = r - 3 = 3 - 3 = 0$

$D_s = D_{si} + D_{se} \Rightarrow 0 + 0 = 0$

Statically determinate.

⑦ (D, ID)



$m = 3,\ j = 3,\ r = 4$

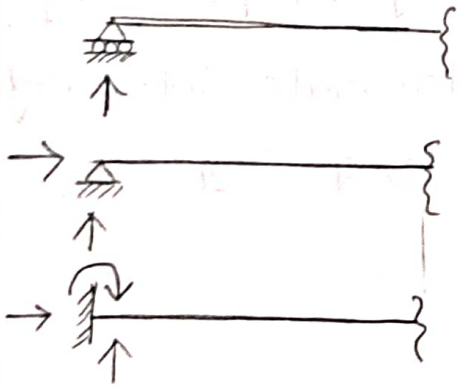
$D_{si} = m - (2j-3) \Rightarrow 3 - (2(3)-3) = 0$

$D_{se} = r - 3 \Rightarrow 4 - 3 = 1$

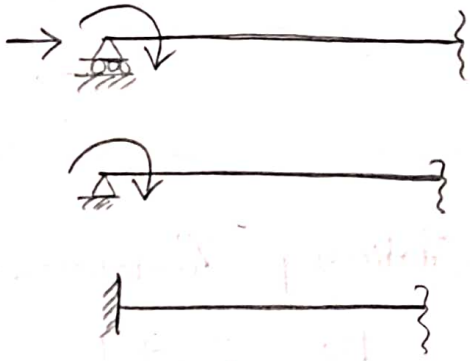
$D_s = D_{si} + D_{se} \Rightarrow 0 + 1 = 1$

Degree of kinematic indeterminacy (D_K) or Degree of freedom (D.O.f):-

Beams :- Constraints



Freedom (D.O.f)



A Structure is said to be kinematically indeterminate if the displacement components of the joints cannot be determined by compatability equations alone. In these structures no: of unknown displacement components are greater than no: of compatability eqns. Hence additional equations based on the equillibrium must be written

in order to obtain sufficient no: of eqn's for the determination of all the unknown displacement Components.

The No: of these additional eqn's necessary for the determination of all independent displacement Components (I.D.C) is known as degree of kinematic indeterminancy. Denoted by D_K.

Beams:

Eg ① :-  $D_K = 4$ (actual)

In general it is assumed that beams & Columns are inextensible.

②  $D_K = 2$ (with assumption)

Kinematically Indeterminate by 2 degree

③  $D_K = 0+0 = 0$

Kinematically determinate.

④  $D_K = 3$

Kinematically Indeterminate by 3rd degree.

⑤  $D_K = 0+1+1 = 2$

Kinematically indeterminate by 2nd degree.

⑥  $D_K = 1+1 = 2$

⑧  $D_K = 0+1 = 1$

⑦ 

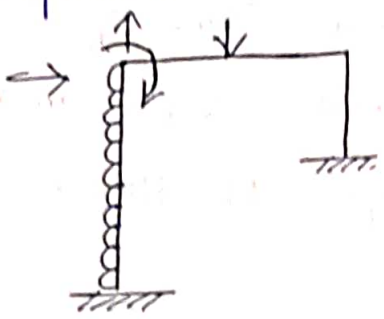
$D_K = 0$ (determinate)

⑨ 

$D_K = 0$.

PORTAL FRAMES :-

Each joint of Portal frame has 3 independent displacement Components i.e, 2 linear and one rotation.



3 · I · D · C

m, r, j

$\therefore D_K = 3j - e$, where e is No: of Compatability equations the No: of equations, of Compatability is equal to no: of Constraints imposed by Support Conditions as each independent External reaction Components Provides the Constraint, the no: of Equations of Compatability is equal to No: of external reactions.

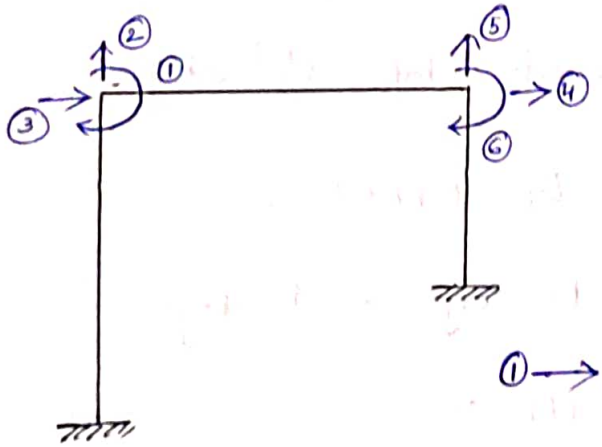
$$D_K = 3j - e$$

$$D_K = 3j - r \text{ (actual)}$$

On the analysis it is generally assumed that beams & Columns are inextensible.

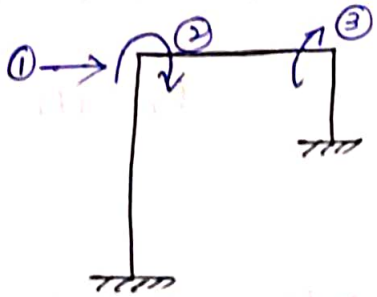
$\therefore D_K = 3j - r - m$ (with assumption).

Eg:-



(without assumption)

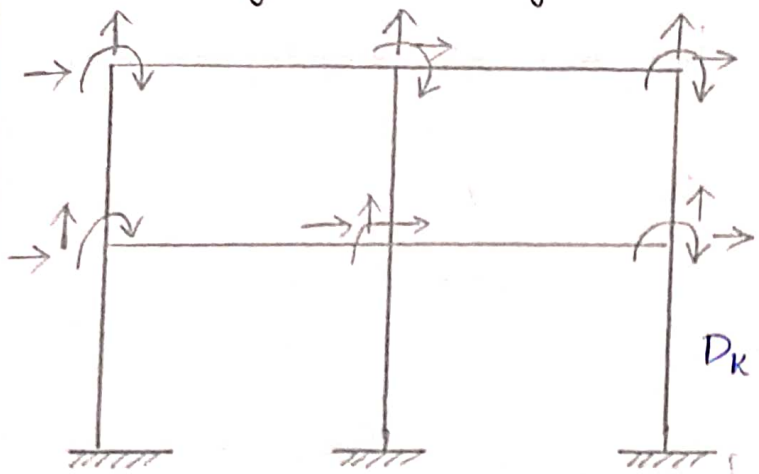
$D_K = 6$.



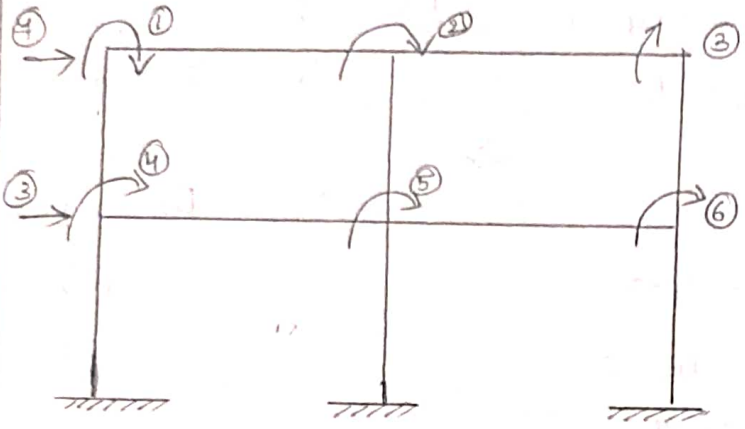
with assumption

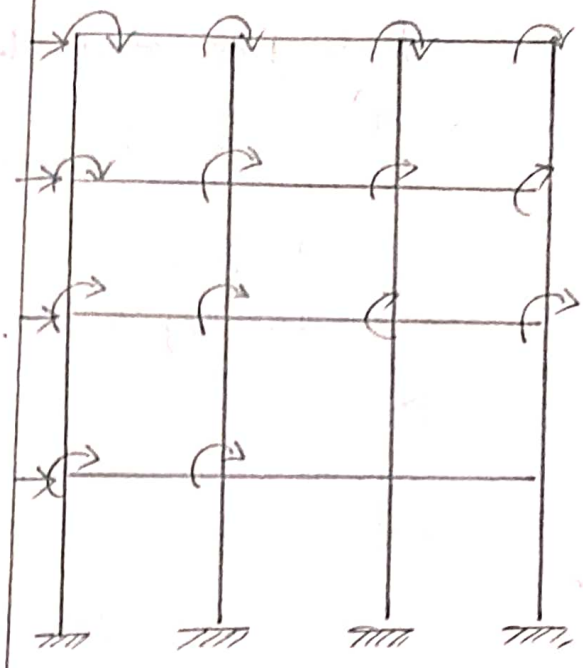
$D_K = 3$

→ Two bay two storeyed (without).



$D_K = 18$.





$D_K = 20$ [With assumption].

$D_K = 3j - r - m \Rightarrow 3(9) - 9 - 10$

$= 8$

For a multi-storey, multi-bay building frame

$D_K = S(B+2)$ ↓ only for fixed bases

S – Storeys

B – Bays.

Shortcut:

→ For Static determinacy for Portal frame

$D_S = 3 \times B \times S \Rightarrow 3 \times 3 \times 4 = 36$

→ $D_S = (3m + r) - 3j$

$D_S = 3(28) + 12 - 3(20)$

$D_S = 36$

Trusses:-

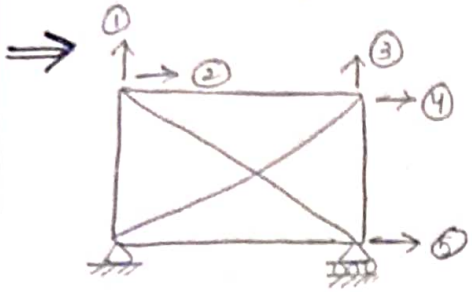
Each joint of a truss has two linear displacement.

∴ for J Joints

Total I.D.C = 2j

$D_K = 2j - e$

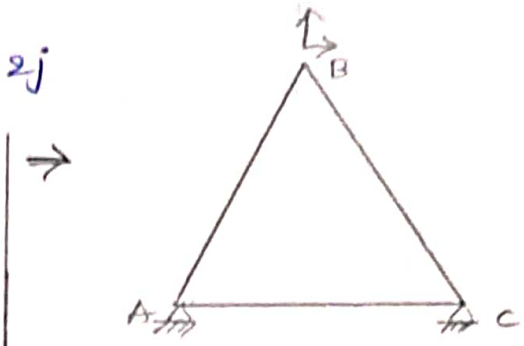
$D_K = 2j - r$



$D_K = 2j - r$

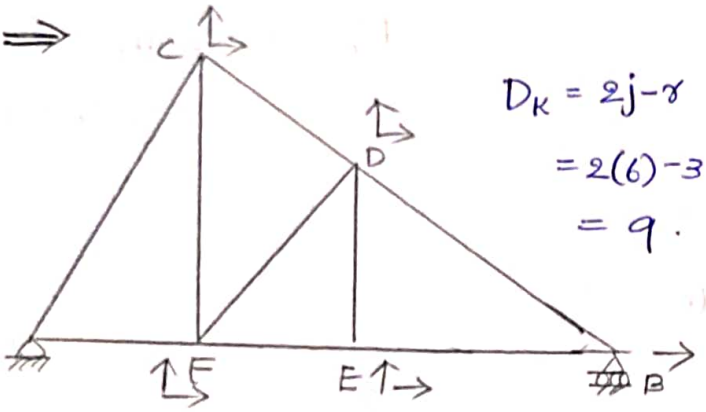
$= 2(4) - 3$

$D_K = 5$



$D_K = 2j - r$

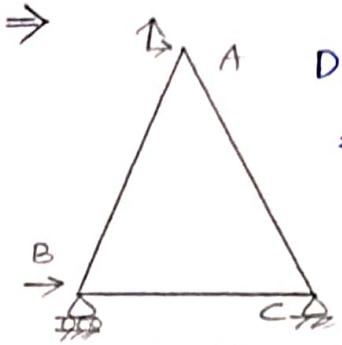
$= 2(3) - 4 = 2.$



$D_K = 2j - r$

$= 2(6) - 3$

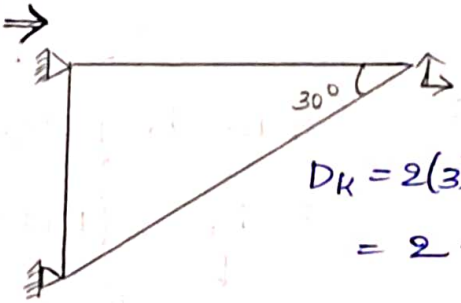
$= 9.$



$D_K = 2j - r$

$= 2(3) - 3$

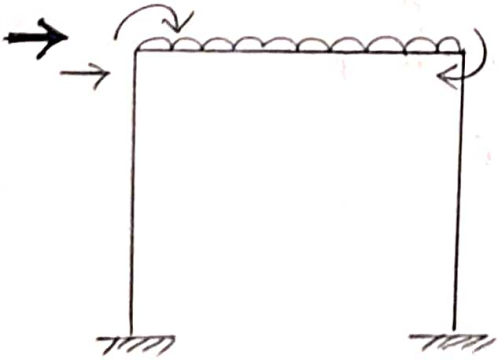
$= 3$



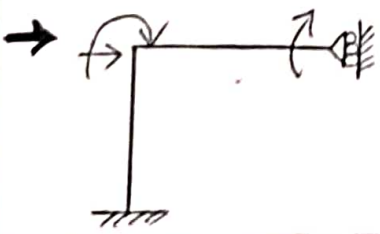
$D_K = 2(3) - 4$

$= 2.$

Portal frames:-

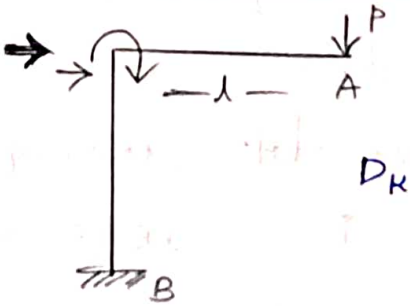


$D_K = 2 + 1 = 3$



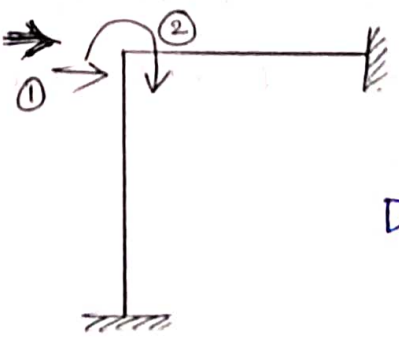
$D_K = 0 + 2 + 1$

$= 3$



$D_K = 0 + 2$

$= 2$



$D_K = 2$

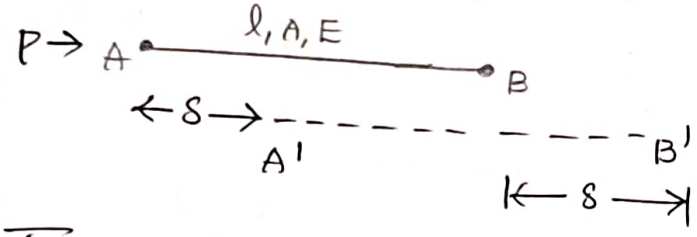
$=$

STIFFNESS:-

It is defined as load required to produce a unit deflection. It is denoted by 'k' and its units are N/mm (or) KN/mm.

Axial Stiffness:-

It is defined as the axial force required to produce a unit axial displacement Consider a bar of length 'AB', length 'l', area 'A', youngs modulus 'E' subjected to an axial load 'P'. Under the application of load the beam AB moves to A'B' & we know.

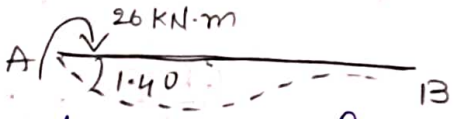


$$\delta = \frac{Pl}{AE}$$

$$\text{Stiffness} = \frac{\text{load}}{\text{deflection}}$$

$$\frac{P}{\delta} = k = \frac{AE}{l}$$

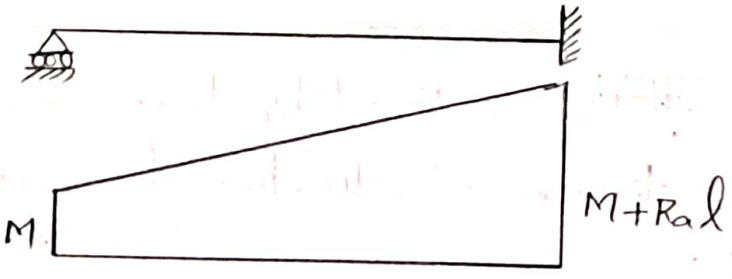
Flexural Stiffness:- (or) Bending:-

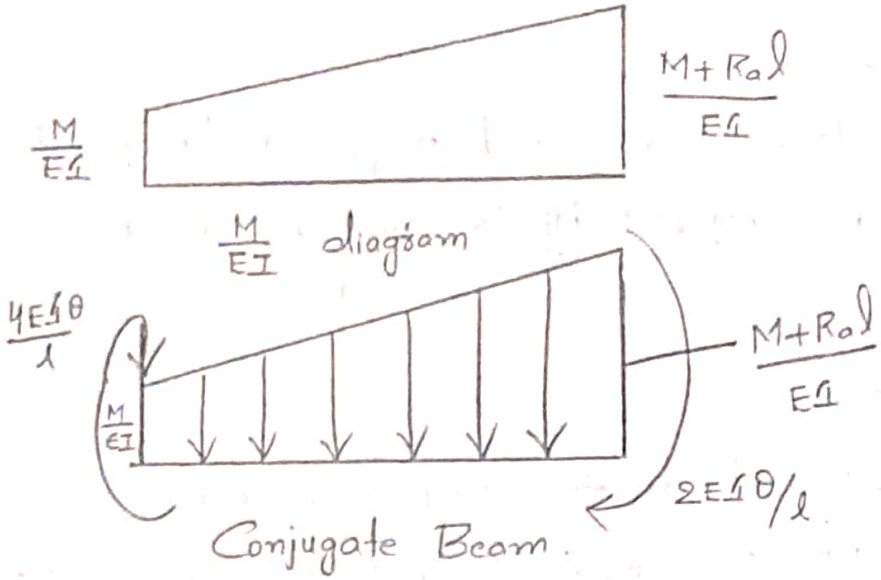


Consider a beam AB whose far end B being fixed and subjected to a moment 'M' at end 'A' as shown in fig below.



Bending (or) flexural Stiffness is defined as the moment required to rotate the end by unity.





The Moment required at End A to rotate it by 'θ' is

$$M_a = \frac{4EI\theta}{l}$$

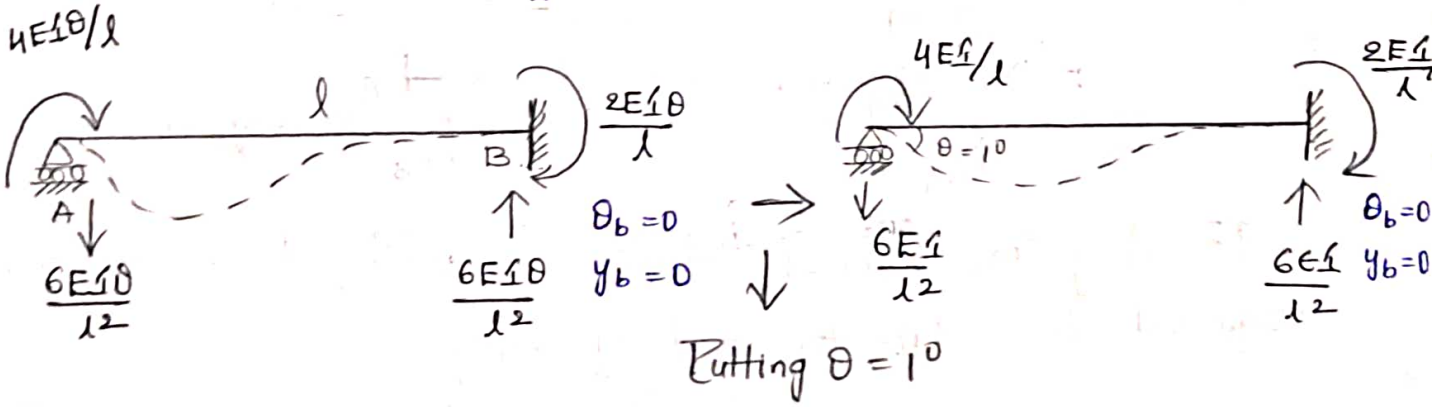
and Moment Produced at End 'B' is $M_b = \frac{2EI\theta}{l}$ in the Same direction.

$$M_B = 0$$

$$\frac{4EI\theta}{l} + R_a l + \frac{2EI\theta}{l} = 0$$

$$\frac{6EI\theta}{l} = -R_a l$$

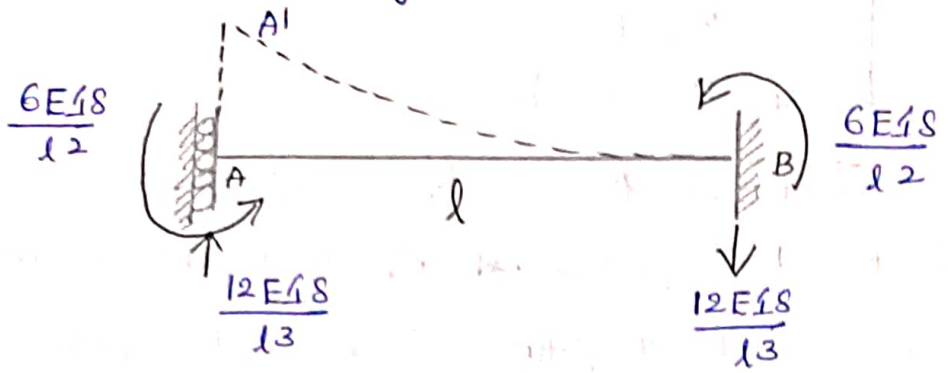
$$\therefore R_a = \frac{-6EI\theta}{l^2}, \quad R_b = \frac{6EI\theta}{l^2}$$



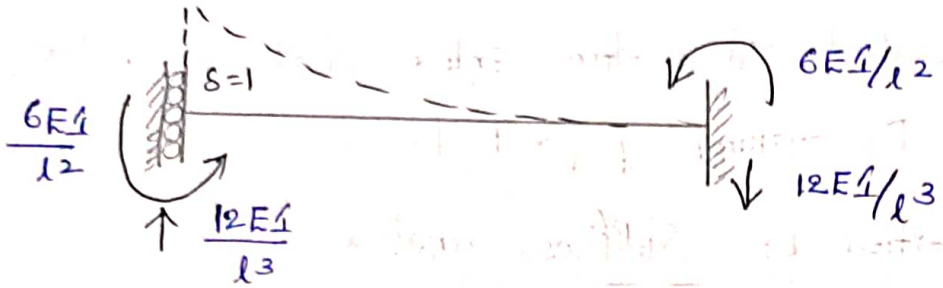
Transverse Stiffness :- is defined as force required to be applied in the transverse direction for unit transverse displacement.

Consider a beam 'AB' whose far End is fixed (B) as

(8) Shown in the fig below.



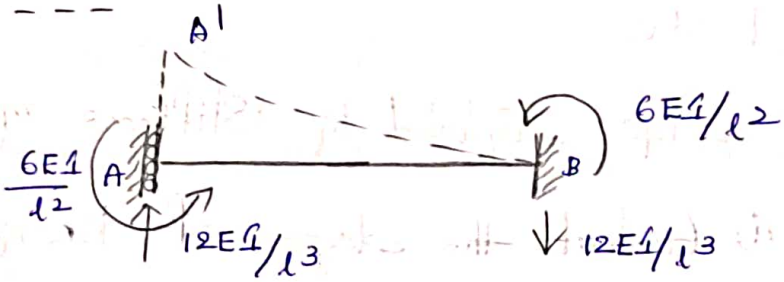
To get the transverse stiffness Put $\Delta = 1$



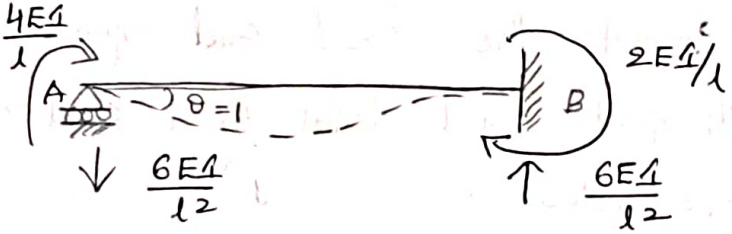
Axial Stiffness:-



Transverse Stiffness:-



Flexural Stiffness:- $\frac{4EI}{l}$



Stiffness method / Displacement Method:-

The stiffness co-efficients in a stiffness matrix is denoted by K_{ij} i.e force at co-ordinate 'i' due to a unit displacement given at the co-ordinate j.

K_{ij} → displacements
↓ force
↓ force / moment.

↳ Axial flexural transverse.

$$[K] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

The first Column of the Stiffness matrix is generated by giving a unit displacement at Co-ordinate 1 and finding Out the forces at all Other Co-ordinates (1, 2, 3, - - -).

Similarly Second, third Column of the Stiffness matrix may be generated. Thus the Entire Stiffness matrix of size Equivalent to D_K making generated.

The Properties of Stiffness matrix :-

a) Stiffness matrix K is a Symmetric matrix i.e $k_{ij} = k_{ji}$.

b) All the diagonal Elements in a Stiffness matrix are Positive.

Steps Involved in Stiffness method :-

① Findout the degree of kinematic Indeterminacy D_K.

② Calculate the fixed End moments due to applied loads, Settlements and total fixed End moments, acting at the Co-ordinates.

③ Generation of Stiffness matrix.

④ Calculation of displacement Components.

$$[\Delta] = [K]^{-1}\{(P) - (P1)\}$$

Where,

$[\Delta]$ = displacement

$[K^{-1}]$ = Stiffness

$[P]$ = loads / moments acting at Coordinates

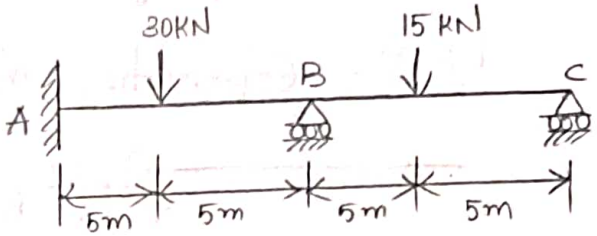
(4) $[P']$ = Total FEM's acting at the Coordinates.

(5) Calculation of final moments by using Slope-deflection Equation.

$$M_{ij} = M_{Fij} + \frac{2EI}{l}\left[2\theta_i + \theta_j - \frac{3\delta}{l}\right]$$

Question:-

①: Analyse the Continuous beam shown in the fig below using Stiffness method.



Sol:- Step ①:- Degree of Kinematic Indeterminacy.

$$D_K = 0+1+1 = 2$$

The Co-ordinates may be assigned as shown in the figure.

Step ②:- FEM's (due to applied load)

$$M'_{AB} = \frac{-wl}{8} = \frac{-30\times10}{8} = -37.5 \text{ kN-m}$$

$$M'_{BA} = \frac{wl}{8} = 37.5 \text{ kN-m}.$$

$$M'_{BC} = \frac{-wl}{8} = \frac{-15\times10}{8} = -18.75 \text{ kN-m}$$

$$M'_{CB} = \frac{wl}{8} = 18.75 \text{ kN-m}.$$

FEM's due to Settlements:-

As there is no yielding

$$M''_{AB} = M''_{BA} = M''_{BC} = M''_{CB} = 0$$

Net FEM's at the Co-ordinates:-

$$P_1' = (M'_{BA} + M'_{BC}) + (M''_{BA} + M''_{BC})$$

$$(37.5 - 18.75) + (0) = 18.75 \text{ kN-m}.$$

$P_2' = M'_{CB} + M''_{CB}$

$= 18.75 + 0 \Rightarrow 18.75$ kN·m.

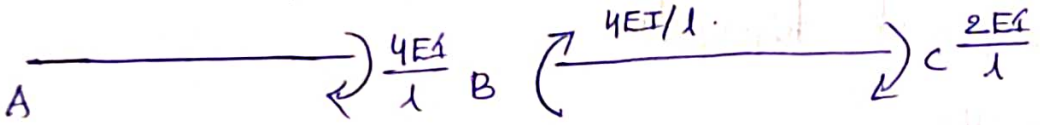
$$P' \begin{bmatrix} P_1' \\ P_2' \end{bmatrix} = \begin{bmatrix} 18.75 \\ 18.75 \end{bmatrix}$$

$$\begin{bmatrix} \theta_b \\ \theta_c \end{bmatrix} = \frac{1}{EI} [$$

[P] = Total moments acting at Co-ordinates

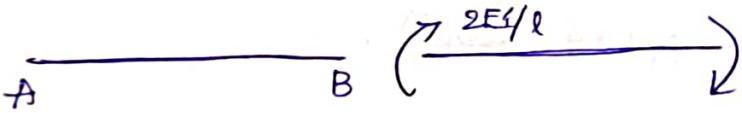
$$[P] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step ③ :- Generation of Stiffness matrix.



$$K_{11} = \frac{4EI}{l} + \frac{4EI}{l} = \frac{4EI}{10} + \frac{4EI}{10} = \frac{8EI}{10} = 0.8EI$$

$$K_{21} = \frac{2EI}{l} = \frac{2EI}{10} = 0.2EI.$$



$$K_{12} = \frac{2EI}{l} = \frac{2EI}{10} = 0.2EI$$

$$K_{22} = \frac{4EI}{l} = \frac{4EI}{10} = 0.4EI$$

$$[K] = EI \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}$$

Step ⑤ :- Displacements :-

$$[\Delta] = [K^{-1}] \{ (P) - (P') \}$$

$$\begin{bmatrix} \theta_b \\ \theta_c \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}^{-1} \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 18.75 \\ 18.75 \end{pmatrix} \right\}$$

$$\begin{bmatrix} \theta_b \\ \theta_c \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}^{-1} \begin{bmatrix} -18.75 \\ -18.75 \end{bmatrix}$$

$$\Rightarrow \frac{1}{EI} \begin{bmatrix} -13.39 \\ -40.18 \end{bmatrix}$$

$$\frac{1}{EI}\begin{bmatrix} -13.39 \\ -40.18 \end{bmatrix}$$

Step ⑥ :- Final Moments

$$M_{AB} = M'_{AB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$$= -37.5 + \frac{2EI}{10}\left[2\times 0 - \frac{13.39}{EI} - 0\right]$$

$$= -40.18 \text{ kN.m}$$

$$M_{BA} = M'_{BA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$$

$$= 37.5 + \frac{2EI}{10}\left[2\times -\frac{13.39}{EI} + 0 - 0\right]$$

$$= 32.144 \text{ kN.m}$$

$$M_{BC} = M'_{BC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$$

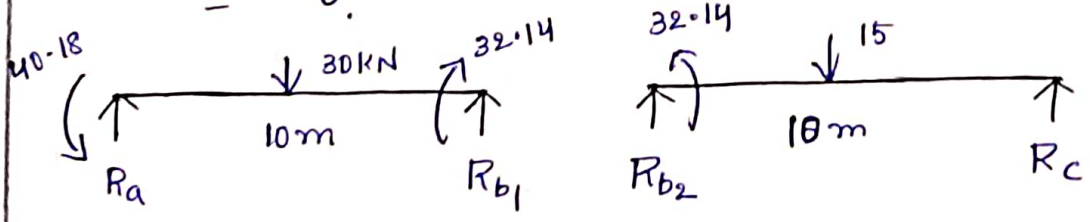
$$= -18.75 + \frac{2EI}{10}\left[2\times -\frac{13.39}{EI} - \frac{40.18}{EI} - 0\right]$$

$$= 32.14 \text{ kN.m}$$

$$M_{CB} = M'_{CB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\delta}{l}\right]$$

$$= 18.75 + \frac{2EI}{10}\left[2\times -\frac{40.18}{EI} - \frac{13.39}{EI} - 0\right]$$

$$= 0$$



$$R_a + R_{b_1} = 30 \text{ kN}$$

CW = A·C·W

$32.14 + (30 \times 5) = 40.18 + R_{b1} \times 10$

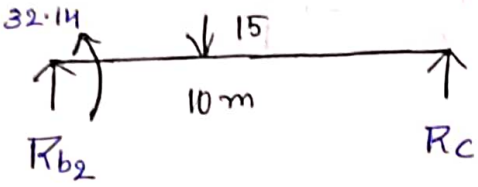
$R_{b1} = 14.196$ kN

$\therefore R_a + R_{b1} = 30$ kN

$R_a = 30 - 14.196$

$= 15.8$ kN.

Next,



$R_{b2} + R_C = 15$ kN

CW = ACW

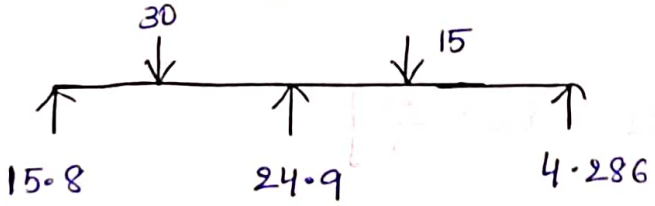
$\Rightarrow 15 \times 5 = 32.14 + R_C \times 10$

$R_C = 4.286$ kN

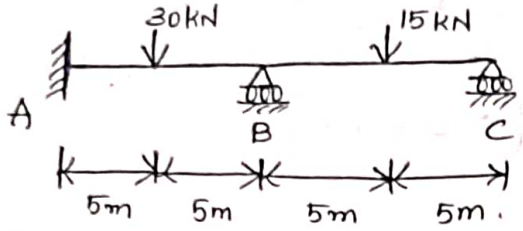
$\therefore R_{b2} + R_C = 15$ kN

$\Rightarrow R_{b2} = 15 - 4.286$

$R_{b2} = 10.714$ kN.



Q:- Analyze the Continuous beam Using Stiffness method. The Supports B and C Settles down by $\frac{250}{EI}$ and $\frac{125}{EI}$ respectively



Sol:- Step ① :- Degree of Kinematic Indeterminacy.

$D_K = 2$

Step ② :- FEM's due to applied load.

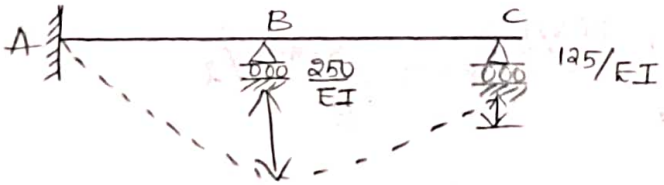
$M'_{AB} = -\frac{wl}{8} = \frac{-30\times10}{8} = -37.5$ kN·m

$M'_{BA} = \frac{wl}{8} = 37.5$ kN·m

$M'_{BC} = -\frac{wl}{8} = -\frac{15\times10}{8} = -18.75$ kN·m

$M'_{CB} = \frac{wl}{8} = 18.75$ kN·m.

(ii) FEM's due to Settlements :-



$\delta \rightarrow$ +Ve - Right

$\delta \rightarrow$ -Ve - left.

$M''_{AB} = M''_{BA} = \frac{-6EI\delta}{l^2}$

$= \frac{-6EI\left(\frac{250}{EI}\right)}{10^2} = -15$ kN·m.

$M''_{BC} = M''_{CB} = \frac{-6EI\delta}{l^2}$

$= \frac{-6EI\left(\frac{-125}{EI}\right)}{10^2} = 7.5$ kN·m

Step③ :- Total FEM's acting @ Co-ordinates.

$$P_1' = (M'_{BA} + M'_{BC}) + (M''_{BA} + M''_{BC})$$

$$= 37.5 + (-18.75) + [-15 + 7.5]$$

$$= 11.25 \text{ kN.m}$$

$$P_2' = M'_{CB} + M''_{CB}$$

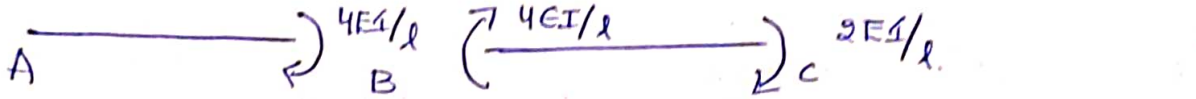
$$= 18.75 + 7.5 = 26.25 \text{ kN.m}$$

$$P' = \begin{bmatrix} P_1' \\ P_2' \end{bmatrix} = \begin{bmatrix} 11.25 \\ 26.25 \end{bmatrix}$$

[P] = Total moments acting at Co-ordinates

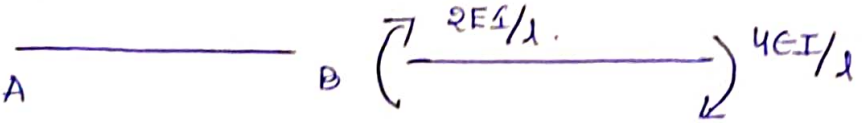
$$[P] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Step④ :- Generation of Stiffness matrix.



$$k_{11} = \frac{4EI}{l} + \frac{4EI}{l} = \frac{4EI}{10} + \frac{4EI}{10} = \frac{8EI}{10} = 0.8EI$$

$$k_{21} = \frac{2EI}{l} = \frac{2EI}{10} = 0.2EI$$



$$k_{12} = \frac{2EI}{l} = \frac{2EI}{10} = 0.2EI$$

$$k_{22} = \frac{4EI}{l} = 4EI/10 = 0.4EI.$$

$$[k] = EI \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}$$

Step⑤ :- Displacements

$$[\Delta] = [k]^{-1} \{[P] - [P']\}$$

$$\begin{bmatrix}\theta_b\\ \theta_c\end{bmatrix} = \frac{1}{EI}\begin{bmatrix}0.8 & 0.2\\ 0.2 & 0.4\end{bmatrix}^{-1}\left\{\begin{bmatrix}0\\0\end{bmatrix} - \begin{bmatrix}11.25\\26.25\end{bmatrix}\right.$$

$$\begin{bmatrix}\theta_b\\ \theta_c\end{bmatrix} = \frac{1}{EI}\begin{bmatrix}0.8 & 0.2\\ 0.2 & 0.4\end{bmatrix}^{-1}\begin{bmatrix}-11.25\\-26.25\end{bmatrix}$$

$$\frac{1}{EI}\begin{bmatrix}2.678\\-66.96\end{bmatrix}$$

-0.1
-8.28

Step ⑥ :- Final Moments :-

$$M_{AB} = M'_{AB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$-37.5 + \frac{2EI}{10}\left(2\times 0 + \frac{2.678}{EI} - \frac{3(250)}{EI\,l}\right) \quad -51$$

$$= -51.96 \text{ KN.m}$$

$$M_{BA} = M'_{BA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$= 37.5 + \frac{2EI}{10}\left(2\times\frac{2.678}{EI} + 0 - \frac{3\times 250}{10EI}\right)$$

$$= 23.57 \text{ KN.m}$$

$$M_{BC} = M'_{BC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$= -18.75 + \frac{2EI}{10}\left[2\times\frac{2.678}{EI} - \frac{66.96}{EI} - \frac{3(-125)}{10EI}\right]$$

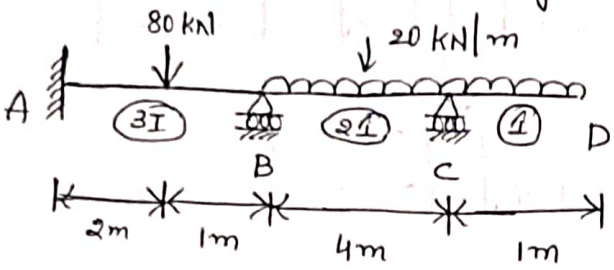
$$= -23.57$$

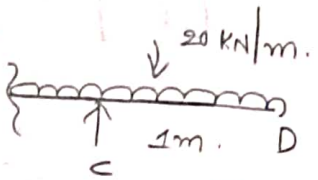
$$M_{CB} = M'_{CB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$$

$$= 18.75 + \frac{2EI}{10}\left[2\times -\frac{66.96}{EI} + \frac{2.678}{EI} - \frac{3(-125)}{10EI}\right]$$

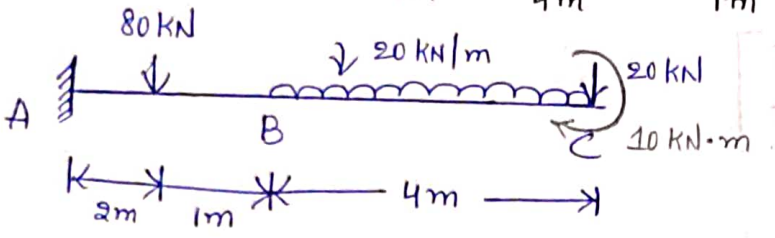
$$= 0$$

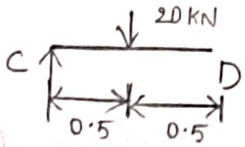
Q:- Analyze the Continous beam by displacement method.

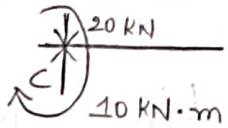




Sol:-







(20×0.5 = 10

Step ①:- Degree of Kinematic Indeterminacy:-

$D_K = 2$

Step ②:- FEM's due to applied load

$$M'_{AB} = \frac{-\omega a b^2}{l^2} = \frac{-80 \times 2 \times 1^2}{3^2} = -17.77 \text{ kN·m}$$

$$M'_{BA} = \frac{\omega a^2 b}{l^2} = \frac{80 \times 2^2 \times 1}{3^2} = 35.55 \text{ kN·m}$$

$$M'_{BC} = \frac{-\omega l^2}{12} = \frac{-20 \times 4^2}{12} = -26.66 \text{ kN·m}$$

$$M'_{CB} = \frac{\omega l^2}{12} = 26.66 \text{ kN·m}.$$

(ii) Settlements:-

$$M''_{AB} = M''_{BA} = M''_{BC} = M''_{CB} = 0$$

Step ③:- Total FEM's acting @ Co-ordinates.

$$P_1' = (M'_{BA} + M'_{BC}) + (M''_{BA} + M''_{BC})$$

$$= 35.55 + (-26.66) + (0+0)$$

$$= 8.89 \text{ kN·m}.$$

3

$P_2' = M'_{CB} + M''_{CB}$

$= 26.66 + 0 = 26.66$ KN-m.

$P' = \begin{bmatrix} P_1' \\ P_2' \end{bmatrix} = \begin{bmatrix} 8.89 \\ 26.66 \end{bmatrix}$

Step ④:- Loads / Moments acting @ Co-Ordinates :-

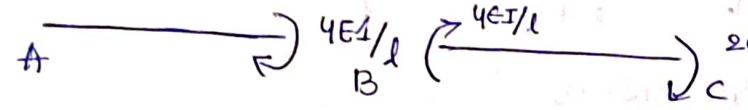
$P_1 = 0$

$P_2 = 10$ KN-m.

∴ the Total moments acting at Co-Ordinates.

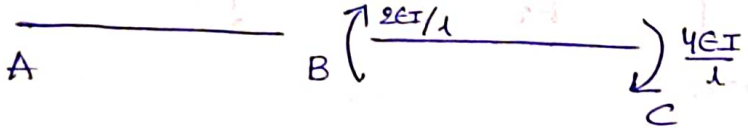
$[P] = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$.

Step ④:- Generation of Stiffness matrix



$K_{11} = \frac{4EI}{l} + \frac{4EI}{l} = \frac{4EI(3)}{3} + \frac{4E(2I)}{4} = 6EI$

$K_{21} = \frac{2EI}{l} = \frac{2E(2I)}{4} = EI$



$K_{12} = \frac{2EI}{l} = \frac{2E(2I)}{4} = EI$

$K_{22} = \frac{4EI}{l} = \frac{4EI(2)}{4} = 2EI$

$[K] = EI\begin{bmatrix} 6 & 1 \\ 1 & 2 \end{bmatrix}$

Step ⑤:- Displacements:-

$[\Delta] = [K]^{-1}\{(P) - (P')\}$

$\begin{bmatrix} \theta_b \\ \theta_c \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 6 & 1 \\ 1 & 2 \end{bmatrix}^{-1}\left\{\begin{bmatrix} 0 \\ 10 \end{bmatrix} - \begin{bmatrix} 8.89 \\ 26.66 \end{bmatrix}\right\}$

$$\frac{1}{EI}\begin{bmatrix} -0.101 \\ -8.279 \end{bmatrix}$$

Step ⑨ :- Final moments

$$M_{AB} = M'_{AB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$$= -17.77 + \frac{2E(3I)}{3}\left[2\times 0 - \frac{0.101}{EI} - 0\right]$$

$$= -17.97 \text{ KN.m}$$

$$M_{BA} = M'_{BA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\Delta}{l}\right]$$

$$= 35.55 + \frac{2E(3I)}{3}\left[2\times - \frac{0.101}{EI} + 0 - 0\right]$$

$$= 35.146 \text{ KN.m}.$$

$$M_{BC} = M'_{BC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$$

$$= -26.66 + \frac{2E(2I)}{4}\left[2\times \frac{-0.101}{EI} - \frac{8.279}{EI} - 0\right]$$

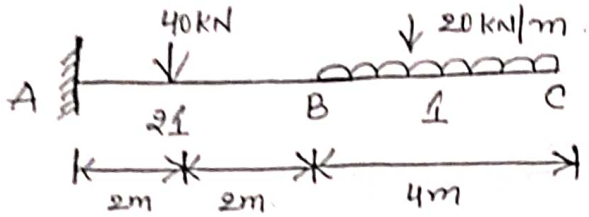
$$= -35.141 \text{ KN.m}.$$

$$M_{CB} = M'_{CB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\delta}{l}\right]$$

$$= 26.66 + \frac{2E(2I)}{4}\left[\frac{2\times -8.27}{EI} - \frac{0.101}{EI} - 0\right]$$

$$= 10.019 \text{ KN.m}$$

Q:- Analyze the Continuous beam by displacement method.



Sol:- Step ① :- Degree of kinematic Indeterminacy :-

$D_K = 2$.

Step ② :- FEM's due to applied load :-

$M'_{AB} = \frac{-\omega l}{8} = -\frac{40 \times 4}{8} = -20 \text{ KN·m}$

$M'_{BA} = \frac{\omega l}{8} = 20 \text{ KN·m}$

$M'_{BC} = \frac{-\omega l^2}{12} = -\frac{20 \times 4^2}{12} = -26.667 \text{ KN·m}$.

$M'_{CB} = \frac{\omega l^2}{12} = 26.67 \text{ KN·m}$.

(ii) Settlements :-

$M''_{AB} = M''_{BA} = M''_{BC} = M'_{CB} = 0$

Step ③ :- Total FEM's acting @ Co-ordinates :-

$P_1' = (M'_{BA} + M'_{BC}) + (M''_{BA} + M''_{BC})$

$= 20 + (-26.67) + (0+0)$

$= -6.67 \text{ KN·m}$.

$P_2' = M'_{CB} + M''_{CB}$

$= 26.67 + 0 = 26.67 \text{ KN·m}$.

$P' = \begin{bmatrix} P_1' \\ P_2' \end{bmatrix} = \begin{bmatrix} -6.67 \\ 26.67 \end{bmatrix}$

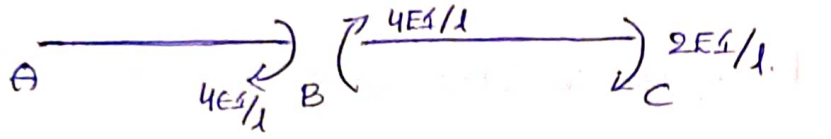
Step ④ :- Loads / moments acting @ Co-ordinates :-

$P_1 = 0$ & $P_2 = 0$

The total moments acting at Co-ordinates

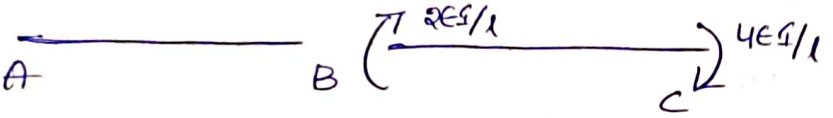
$$[P] = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step ④:- Generation of Stiffness matrix:-



$$K_{11} = \frac{4EI}{l} + \frac{4EI}{l} = \frac{4E(2I)}{4} + \frac{4EI}{4} = 3EI$$

$$K_{21} = \frac{2EI}{l} = \frac{2E(I)}{4} = 0.5\,EI$$



$$K_{12} = \frac{2EI}{l} = \frac{2EI}{4} = 0.5\,EI$$

$$K_{22} = \frac{4EI}{l} = \frac{4EI}{4} = EI$$

$$[K] = EI \begin{bmatrix} 3 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

Step ⑤:- Displacements:-

$$[\Delta] = [K^{-1}]\{(P) - (P')\}$$

$$\begin{bmatrix} \theta_b \\ \theta_c \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 3 & 0.5 \\ 0.5 & 1 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -6.67 \\ 26.67 \end{bmatrix} \right\}$$

$$\frac{1}{EI} \begin{bmatrix} 7.27 \\ -30.30 \end{bmatrix}$$

Step ⑦:- final Moments:-

$$M_{AB} = M'_{AB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$$= -20 + \frac{2E(2I)}{4}\left[2 \times 7.27\,0 + \frac{7.27}{EI} - 0\right] = -12.73\ kN\cdot m$$

$$M_{BA} = M'_{BA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$$

$$= 20 + \frac{2E(2I)}{4}\left[2\times\frac{7.27}{EI} + 0 - 0\right]$$

$$= 34.54 \text{ kN.m}$$

$$M_{BC} = M'_{BC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$$

$$= -26.67 + \frac{2EI}{4}\left[2\times\frac{7.27}{EI} - \frac{30.30}{EI} - 0\right]$$

$$= -34.55 \text{ kN.m}$$

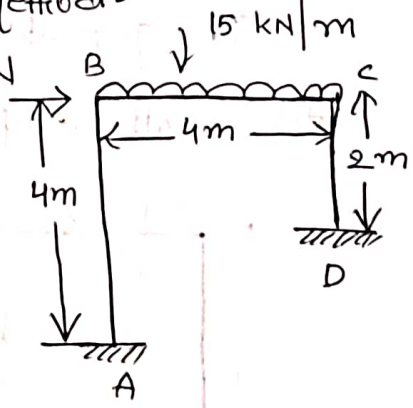
$$M_{CB} = M'_{CB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\delta}{l}\right]$$

$$= 26.67 + \frac{2EI}{4}\left[2\times\frac{-30.30}{EI} + \frac{7.27}{EI} - 0\right]$$

$$= 0$$

PORTAL FRAMES:-

Q:- Analyze the Portal frames by Stiffness method.



Sol:- Step ① :- Degree of kinematic Indeterminacy.

$$D_k = 3j - r - m$$

$$= 3(4) - 6 - 3 \Rightarrow 3$$

Step ② :- FEM's due to applied loads :-

$$M'_{AB} = M'_{BA} = M'_{CD} = M'_{DC} = 0$$

$$M'_{BC} = \frac{-wl^2}{12} = \frac{-15\times 4^2}{12} = -20 \text{ kN.m}$$

$$M'_{CB} = 20 \text{ kN.m}$$

(ii) FEM's due to Settlements.

$M''_{AB} = M''_{BA} = M''_{BC} = M''_{CB} = M''_{CD} = M''_{DC} = 0$.

Step ③ :- Total FEM's acting @ the Co-ordinates

$P_1' = 0$

$P_2' = (M'_{BA} + M'_{BC}) + (M''_{BA} + M''_{BC})$

$= (0 - 20) + 0 = -20$ kN-m.

$P_3' = (M'_{CB} + M'_{CD}) + (M''_{CB} + M''_{CD})$

$= (20 + 0) = 20$ kN-m.

Step ④ :- Loads/moments acting at the Coordinates.

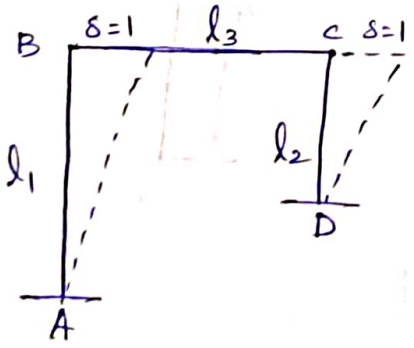
$P_1 = 25$ kN

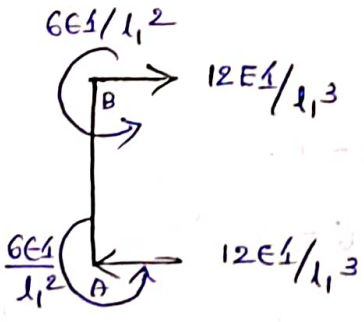
$P_2 = 0$ & $P_3 = 0$

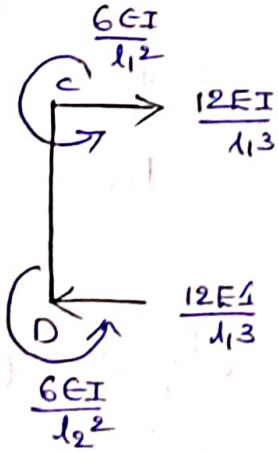
$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 25 \\ 0 \\ 0 \end{bmatrix} = [P]$$

Step ⑤ :- Generation of stiffness matrix.

To generate the first Column of Stiffness matrix give the Unit displacement at Coordinate 1.







$K_{11} = \frac{12EI}{l_1^3} + \frac{12EI}{l_2^3}$

$= \frac{12EI}{4^3} + \frac{12EI}{2^3}$

$= \boxed{1.68\,EI}$

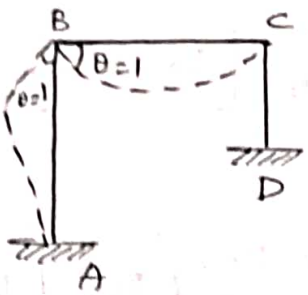
$K_{21} = -\frac{6EI}{l_1^2}$

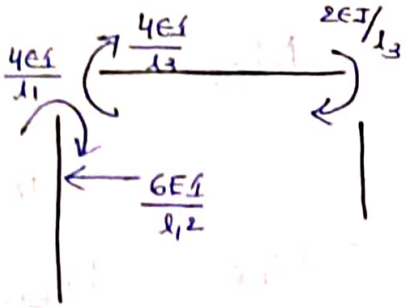
$= \boxed{-0.375\,EI}$

$K_{31} = \frac{-6EI}{l_2^2} = \boxed{-1.5\,EI}$

$$K_{12} = \frac{-6EI}{l_1^2} = \frac{-6EI}{4^2}$$

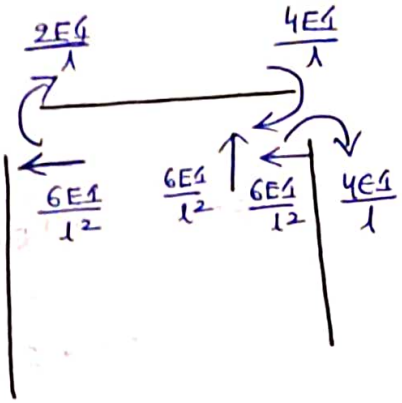
$$= -0.375EI$$





$$K_{22} = \frac{4EI}{l_3} + \frac{4EI}{l_1} \Rightarrow \frac{4EI}{4} + \frac{4EI}{4} = 2EI$$

$$K_{32} = \frac{2EI}{l_3} = \frac{2EI}{4} = 0.5EI$$



$$K_{13} = \frac{-6EI}{l_1^2} = \frac{-6EI}{4^2} = 1.5EI$$

$$K_{23} = \frac{2EI}{l_3} = \frac{2EI}{4} = 0.5EI$$

$$K_{33} = \frac{4EI}{l_3} + \frac{4EI}{l_2} = \frac{4EI}{4} + \frac{4EI}{2} = 3EI$$

$$\therefore [K] = EI \begin{bmatrix} 1.68 & -0.375 & 1.5 \\ -0.375 & 2 & 0.5 \\ -1.5 & 0.5 & 3 \end{bmatrix}$$

Step ⑥ :- Displacements.

$$[\Delta] = [K]^{-1}\{[P] - [P']\}$$

$$\frac{1}{EI}\begin{bmatrix} 1.68 & -0.375 & 1.5 \\ -0.375 & 2 & 0.5 \\ -1.5 & 0.5 & 3 \end{bmatrix}^{-1}\left\{\begin{bmatrix} 25 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -20 \\ 20 \end{bmatrix}\right\}$$

$$\begin{bmatrix} \delta \\ \theta_b \\ \theta_c \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 17.91 \\ 13.29 \\ 0 \end{bmatrix}$$

Step ⑦ Final Moments

$$M_{AB} = M'_{AB} + \frac{2EI}{l_1}\left[2\theta_a + \theta_b - \frac{3\delta}{l}\right]$$

$$M_{AB} = 0 + \frac{2EI}{4}\left[2\times 0 + \frac{13\cdot 2}{EI} - \frac{3\times 17\cdot 91}{4}\right]$$

$$= 0.$$

$$M_{BA} = M'_{BA} + \frac{2EI}{4}\left[2\theta_b + \theta_A - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2EI}{4}\left[2\times\frac{13\cdot 29}{EI} + 0 - \frac{3\times 17\cdot 91}{4}\right]$$

$$= 6\cdot 57 \text{ kN.m}.$$

$$M_{BC} = M'_{BC} + \frac{2EI}{l_3}\left[2\theta_b + \theta_C - \frac{3\delta}{l}\right]$$

$$= -20 + \frac{2EI}{4}\left[2\times\frac{13\cdot 29}{EI} + 0 - 0\right] = -6\cdot 71 \text{ kN.m}.$$

$$M_{CB} = M'_{CB} + \frac{2EI}{l}\left[2\theta_C + \theta_b - \frac{3\delta}{l}\right]$$

$$= 20 + \frac{2EI}{4}\left[2\times 0 + \frac{13\cdot 29}{EI} - 0\right] = 26\cdot 645 \text{ kN.m}.$$

$$M_{CD} = M'_{CD} + \frac{2EI}{l}\left[2\theta_C + \theta_D - \frac{3\delta}{l}\right]$$

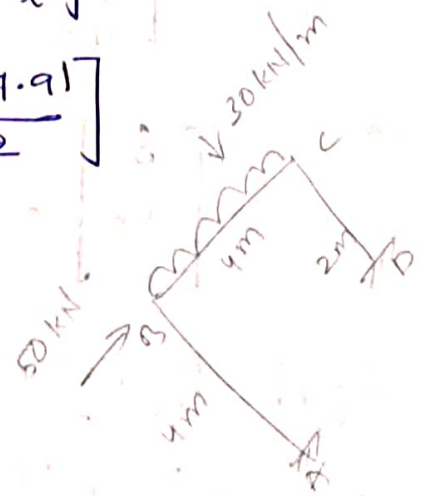
$$= 0 + \frac{2EI}{2}\left[2\times 0 + 0 - \frac{3\times 17\cdot 91}{2}\right]$$

$$= -26\cdot 865 \text{ kN.m}.$$

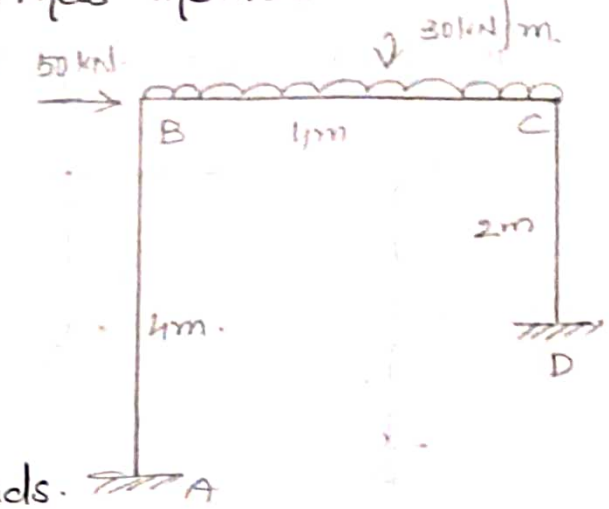
$$M_{DC} = M'_{DC} + \frac{2EI}{2}\left[2\theta_D + \theta_C - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2EI}{2}\left[2\times 0 + 0 - \frac{3\times 17\cdot 91}{2}\right]$$

$$= -26\cdot 86 \text{ kN.m}.$$



Q:- Analyze the Portal frame by Stiffness method.



Sol:- Step ① :- Degree of kinematic Indeterminacy.

$D_k = 3j - r - m$

$= 3(4) - 6 - 3 \Rightarrow 3$.

Step ② :- FEM's due to applied loads.

$M'_{AB} = M'_{BA} = M'_{CD} = M'_{DC} = 0$

$M'_{BC} = \frac{-wl^2}{12} = \frac{-15 \times 30 \times 4^2}{12} = -40 \text{ kN.m}$

$M'_{CB} = 40 \text{ kN.m}$.

(ii) FEM's due to Settlements

$M''_{AB} = M''_{BA} = M''_{BC} = M''_{CB} = M''_{CD} = M''_{DC} = 0$.

Step ③ :- Total FEM's acting at the Co-ordinates.

$P_1' = 0$

$P_2' = (M'_{BA} + M'_{BC}) + (M''_{BA} + M''_{BC})$

$= (0 - 40) + 0 = -40 \text{ kN.m}$.

$P_3' = (M'_{CB} + M'_{CD}) + (M''_{CB} + M''_{CD})$

$= (40 + 0) = 40 \text{ kN.m}$.

Step ④ :- Loads / moments acting at the Co-ordinates

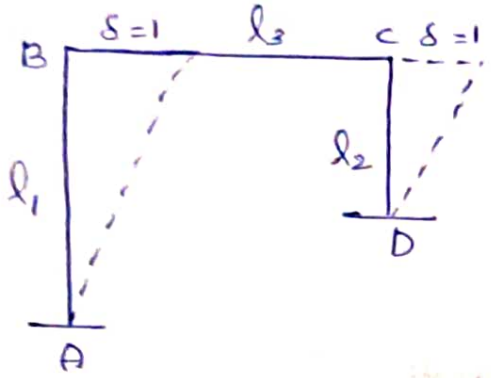
$P_1 = 250 \text{ kN}$

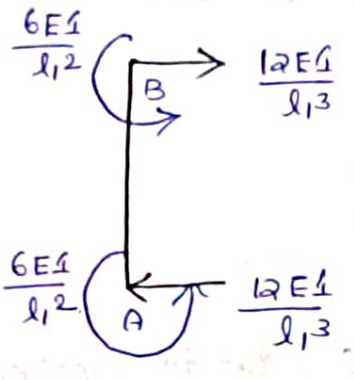
$P_2 = 0 \quad \& \quad P_3 = 0$

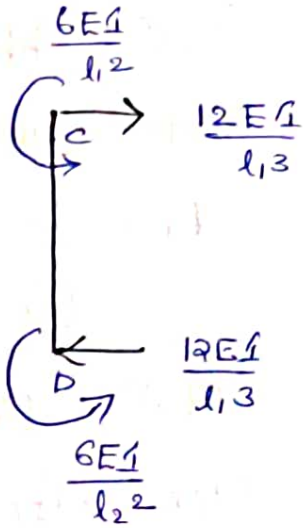
$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix} = [P]$$

Step ⑤ :- Generation of Stiffness matrix.

To Generate the first Column of Stiffness matrix give the Unit displacement at Coordinate 1.







$$K_{11} = \frac{12EI}{l_1^3} + \frac{12EI}{l_2^3}$$

$$= \frac{12EI}{4^3} + \frac{12EI}{2^3}$$

$$= 1.68\ EI$$

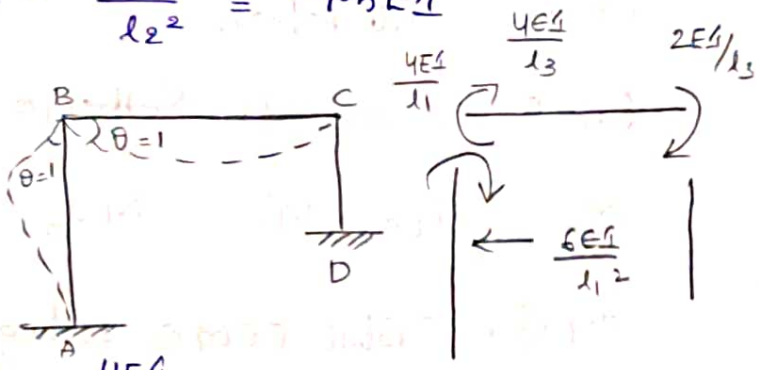
$$K_{21} = -\frac{6EI}{l_1^2}$$

$$= -0.375\ EI$$

$$K_{31} = -\frac{6EI}{l_2^2} = -1.5EI$$

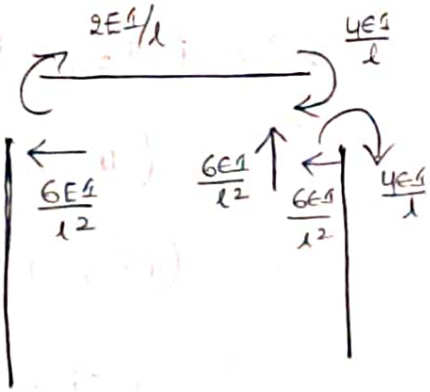
$$K_{12} = -\frac{6EI}{l_1^2} = -\frac{6EI}{4^2}$$

$$= -0.375\ EI$$



$$K_{22} = \frac{4EI}{l_3} + \frac{4EI}{l_1} \Rightarrow \frac{4EI}{4} + \frac{4EI}{4} = 2EI.$$

$$K_{32} = \frac{2EI}{l_3} = \frac{2EI}{4} = 0.5EI$$



$$K_{13} = -\frac{6EI}{l_2^2} = -\frac{6EI}{2^2} = 1.5EI$$

$$K_{23} = \frac{2EI}{l_3} = \frac{2EI}{4} = 0.5EI$$

$$K_{33} = \frac{4EI}{l_3} + \frac{4EI}{l_2} = 3EI.$$

$$[K] = EI \begin{bmatrix} 1.68 & -0.375 & 1.5 \\ -0.375 & 2 & 0.5 \\ -1.5 & 0.5 & 3 \end{bmatrix}$$

Step ⑥ :- Displacements

$$[\Delta] = [K]^{-1}\{[P] - [P']\}$$

$$\frac{1}{EI}\begin{bmatrix} 1.68 & -0.375 & 1.5 \\ -0.375 & 2 & 0.5 \\ -1.5 & 0.5 & 3 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -40 \\ 40 \end{bmatrix} \right\}$$

$$\begin{bmatrix} \delta \\ \theta_b \\ \theta_c \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 35.667 \\ 26.673 \\ 0 \end{bmatrix}$$

Step ⑦ final Moments

$$M_{AB} = M'_{AB} + \frac{2EI}{l}\left[2\theta_a + \theta_b - \frac{3\delta}{l}\right]$$

$$M_{AB} = 0 + \frac{2EI}{4}\left[2\times 0 + \frac{26.67}{EI} - \frac{3\times 35.66}{4}\right]$$

$$= 0$$

$$M_{BA} = M'_{BA} + \frac{2EI}{4}\left[2\theta_b + \theta_A - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2EI}{4}\left[2\times\frac{26.67}{EI} + 0 - \frac{3\times 35.66}{4}\right]$$

$$= 13.29 \text{ kN·m}$$

$$M_{BC} = M'_{BC} + \frac{2EI}{l}\left[2\theta_b + \theta_c - \frac{3\delta}{l}\right]$$

$$= -40 + \frac{2EI}{4}\left[2\times\frac{26.673}{EI} + 0 - 0\right]$$

$$= -13.32 \text{ kN·m}$$

$$M_{CB} = M'_{CB} + \frac{2EI}{l}\left[2\theta_c + \theta_b - \frac{3\delta}{l}\right]$$

$$= 40 + \frac{2EI}{4}\left[2\times 0 + \frac{26.67}{EI} - 0\right]$$

$$= 53.33 \text{ kN·m}$$

$$M_{CD} = M'_{CD} + \frac{2EI}{l}\left[2\theta_C + \theta_D - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2EI}{2}\left[2\times 0 + 0 - \frac{3\times 35.66}{2}\right]$$

$$= -53.49 \text{ kN.m}$$

$$M_{DC} = M'_{DC} + \frac{2EI}{2}\left[2\theta_D + \theta_C - \frac{3\delta}{l}\right]$$

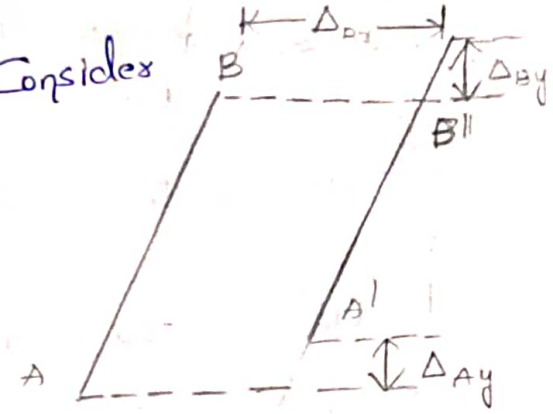
$$= 0 + \frac{2EI}{2}\left[2\times 0 + 0 - \frac{3\times 35.66}{2}\right]$$

$$= -53.49 \text{ kN.m}.$$

TRUSS [Stiffness method]

displacement $[\Delta] = [K]^{-1}[P]$

Consider a truss member AB which has been moved to A'B' upon the application of load as shown in the fig. below



Stiffness matrix for truss Element Consider as

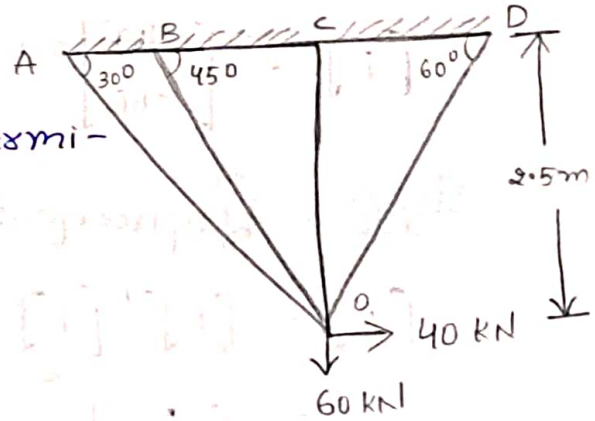
$$[k] = \begin{bmatrix} \cos^2\theta & \cos\theta \cdot \sin\theta \\ \cos\theta \cdot \sin\theta & \sin^2\theta \end{bmatrix} \times \frac{AE}{l}$$

Where θ = angle measuremed from $\leftarrow \Delta_{Ax} \rightarrow$ the horizontal line to a particular member in anticlock wise direction.

The member forces may be found Out Using the formula

$$F_{AB} = \frac{AE}{l}\left[(\Delta_{Bx} - \Delta_{Ax})\cos\theta_{AB} + (\Delta_{By} - \Delta_{Ay})\sin\theta_{AB}\right].$$

Q:- Analyze the truss Shown in the figure below by Stiffness method. The Cross-Sectional area of all the members is $1000\ mm^2$. Take $E = 200\ kN/mm^2$



Sol:- Step (i): Degree of kinematic Indeterminacy

$D_k = 2$

Length of members

$$\sin 30^\circ = \frac{OC}{AO} = \frac{2.5}{OA}$$

$OA = 5m$.

(ii) $\sin 45^\circ = \frac{2.5}{OB} \Rightarrow OB = 3.53\ m$

(iii) $\sin 60^0 = \frac{2.5}{OD}$

$\Rightarrow OD = 2.88\ m$.

member	length (m)	Angle (θ)	$\frac{AE}{l}$ KN/mm	$\frac{AE}{l}\cos^2\theta$	$\frac{AE}{l}\cos\theta \cdot \sin\theta$	$\frac{AE}{l}\sin^2\theta$
OA	5	150^0	40	30	-17.32	10
OB	3.53	135^0	56.67	28.32	-28.33	28.33
OC	2.5	90^0	80	0	0	80
OD	2.88	60^0	69.44	17.36	30.068	52.08
			$\Sigma AE/l$ = ~~75.68~~ 246.11	75.68	-15.58	170.41

Step (2) :- Stiffness matrix of Truss Element :-

$$[K] = \frac{AE}{l}\begin{bmatrix} \cos^2\theta & \cos\theta \cdot \sin\theta \\ \cos\theta \cdot \sin\theta & \sin^2\theta \end{bmatrix} = \begin{bmatrix} 75.68 & -15.6 \\ -15.6 & 170.41 \end{bmatrix}$$

$$[P] = \begin{bmatrix} 40 \\ -60 \end{bmatrix}$$

Step (3) : Displacements.

$$[\Delta] = [K]^{-1}[P]$$

$$= \begin{bmatrix} \Delta_{Ox} \\ \Delta_{Oy} \end{bmatrix} = \begin{bmatrix} 0.46 \\ -0.30 \end{bmatrix}$$

Member forces : may be found out by using formula

$$F_{AB} = AE/l\left[(\Delta_{Bx} - \Delta_{Ax})\cos\theta_{AB} + (\Delta_{By} - \Delta_{Bx})\sin\theta_{AB}\right]$$

8

$$F_{OA} = \frac{AE}{l}\left[(\Delta_{Ax} - \Delta_{Ox})\cos\theta_{OA} + (\Delta_{Ay} - \Delta_{Oy})\sin\theta_{OA}\right]$$

$$= 40\left[(0 - 0.46)\cos 150^0 + (0 - (-0.30))\sin 150^0\right]$$

$$= 21.93 \text{ kN}.$$

$$F_{OB} = 56.67\left[(0 - 0.46)\cos 135^0 + (0 + 0.30)\sin 135^0\right] = 30.45 \text{ kN}$$

$$F_{OC} = 80\left[(0 - 0.46)\cos 90^0 + (0 + 0.30)\sin 90^0\right] = 24 \text{ kN}$$

$$F_{OD} = 69.44\left[(0 - 0.46)\cos 60^0 + (0 + 0.30)\sin 60^0\right] = 2.06 \text{ kN}.$$

Flexibility method :-

This method is also called as "force method", redundant forces are treated as unknowns. It is also called as compatability method since the conditions of compatability of displacements is utilized.

δ_{ij} → unit force

formulation of flexibility matrix :-

The flexibility co-effecients is denoted by "δ_{ij}" i.e displacement at co-ordinate 'i' due to a unit load at 'j' and is given by $\delta_{ij} = \int_0^l \frac{m_i m_j}{EI} \cdot dx$

To generate the first column of flexibility matrix apply a unit force (load/moment) and find out the displacements.

→ The unknown redundant forces (reactions/moments) may be computed using the formula

$$[P] = [\delta]^{-1}\{(\Delta) - (\Delta_L)\}$$

Where,

δ = flexibility matrix

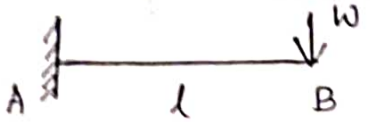
P = redundant forces

Δ = displacements of sinkings rotates left or right.

Δ_L = displacements due to applied loads.

The other member forces may be found out by using the Equations of Equillibrium.

Date:- 02/01/2023

1) 

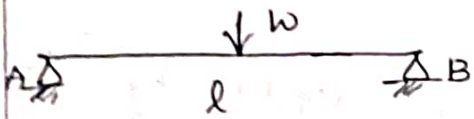
$\theta_a = 0$, $\theta_b = \frac{wl^2}{2EI}$ (Slope)

$\Delta_a = 0$, $\Delta_b = \frac{wl^3}{3EI}$

2) 

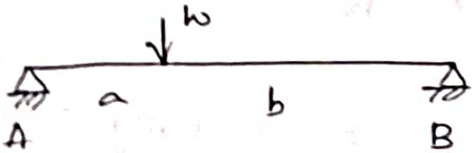
$\theta_a = 0$ $\theta_b = \frac{wl^3}{6EI}$

$\Delta_a = 0$, $\Delta_b = \frac{wl^4}{8EI}$

3) 

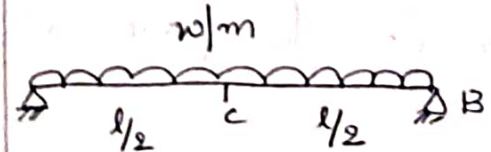
$\theta_a = \theta_b = \frac{wl^2}{16EI}$

$\Delta_A = \Delta_B = 0$ & $\Delta_C = \frac{wl^3}{48EI}$.

4) 

$\theta_a = \frac{wab(2l-a)}{6lEI}$ $\Delta_C = \frac{wa^2b^2}{3lEI}$

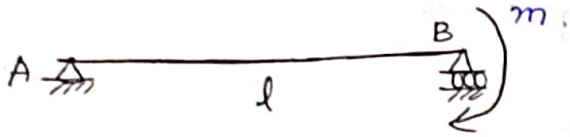
$\theta_b = \frac{wa(l^2-a^2)}{6lEI}$. $\Delta_a = \Delta_b = 0$

5) 

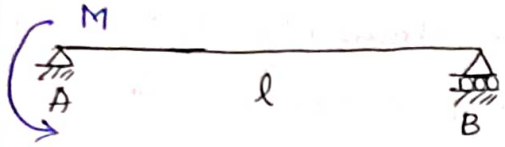
$\theta_a = \theta_b = \frac{wl^3}{24EI}$ $\Delta_C = \frac{5wl^4}{384EI}$

$\Delta_a = \Delta_b = 0$.

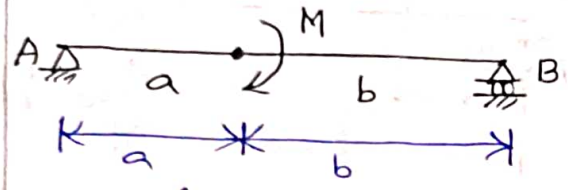
Standard Results :-

6) 

$\theta_a = \frac{-Ml}{6}$, $\theta_b = \frac{Ml}{3}$

7) 

$\theta_a = \frac{Ml}{3}$ $\theta_b = \frac{-Ml}{6}$.

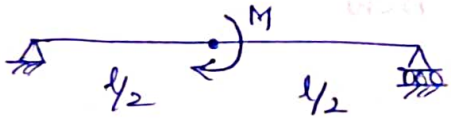
8) 

$\theta_a = \frac{M}{6lEI}(3b^2 - l^2)$

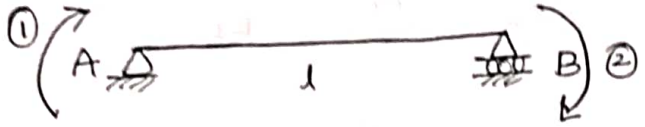
$\theta_b = \frac{M}{6lEI}[2l^2 + 3b^2 - 6bl]$.

Note :- If moment acts at the Centre i-e $a = b = l/2$

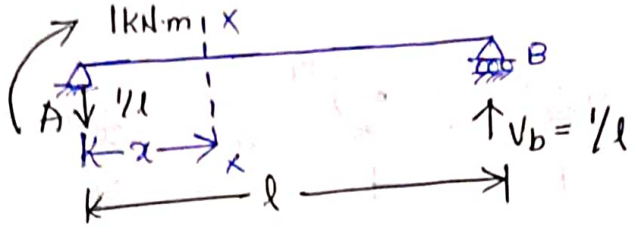
$\theta_a = \theta_b = \frac{Ml}{24EI}$



Q:- Formulate the flexibility matrix for the beam having the Co-ordinates as shown in the figure below



To generate the first Column of flexibility matrix apply a Unit moment 1KN-m at Co-ordinate 1 as shown in the figure.



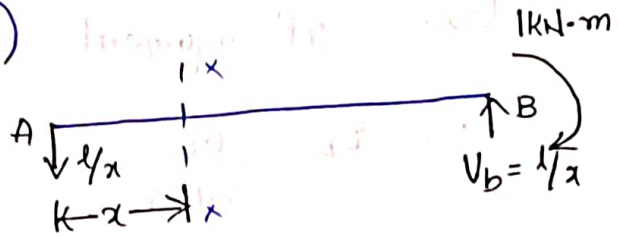
$\Sigma M_A = 0$, CW = ACW

$1 = V_b \times l$

$V_b = 1/l$

$V_a + V_b = 0$ Therefore $m_1 = m_x = -\frac{1}{l}x + 1$

$\therefore V_a = -1/l.$ $= (1 - \frac{x}{l})$



$\Sigma M_A = 0$, CW = ACW

$1 = V_b \times l$

$\therefore \boxed{V_b = 1/l}$

$V_a + V_b = 0$ Therefore, $m_2 = m_x = \frac{-1}{l} \times x = \frac{-x}{l}$

$\therefore \boxed{V_a = -1/l}$

Step ②:- Flexibility Co-effecient.

$$\delta_{ij} = \int_0^l \frac{m_i m_j}{EI} \cdot dx.$$

$$\therefore \delta_{11} = \int_0^l \frac{m_1 \times m_1}{EI} \cdot dx = \int_0^l \frac{(1 - x/l)(1 - x/l)}{EI} \cdot dx.$$

$$= \int_0^l \frac{(1 - x/l)^2}{EI} \cdot dx \Rightarrow \left[\frac{(1 - x/l)^3}{3(0 - 1/l)EI} \right]_0^l$$

$$\Rightarrow \frac{1}{3EI} \left[-l(1 - x/l)^3 \right]_0^l \Rightarrow \frac{-l}{3EI}[0 - 1] \Rightarrow \frac{l}{3EI}.$$

$$\delta_{21} = \delta_{12} = \int_0^{l} \frac{m_1 \cdot m_2}{EI} \cdot dx$$

$$= \int_0^{l} \frac{(1 - x/l)\left(\frac{-x}{l}\right)}{EI} \cdot dx \Rightarrow \int_0^{l} \frac{\left(\frac{-x}{l} + \frac{x^2}{l^2}\right)dx}{EI}$$

$$= \frac{1}{EI}\left[\left[\frac{-x^2}{2l} + \frac{x^3}{3l^2}\right]\right]_0^{l} \Rightarrow + \frac{1}{EI}\left[\frac{-l^2}{2l} + \frac{l^3}{3l^2}\right] - 0$$

$$\frac{1}{EI}\left[\frac{-l}{2} + \frac{l}{3}\right] \Rightarrow \frac{-l}{6EI}$$

$$\delta_{22} = \int_0^{l} \frac{m_2 \cdot m_2}{EI} \cdot dx$$

$$= \int_0^{l} \frac{(-x/l)(-x/l)}{EI} \cdot dx$$

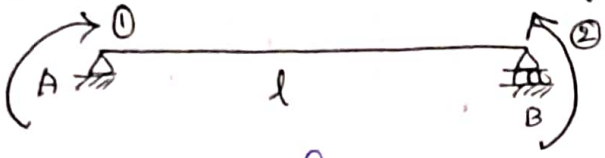
$$= \frac{1}{EI\, l^2}\left[\frac{x^3}{3}\right]_0^{l} \Rightarrow \frac{l}{3EI}$$

$$[\delta] = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

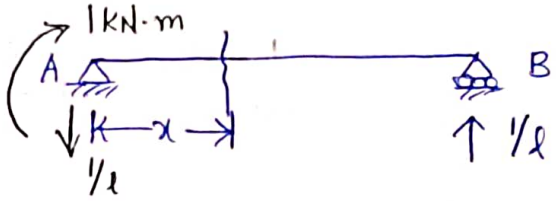
$$= \begin{bmatrix} l/3EI & -l/6EI \\ -l/6EI & l/3EI \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} l/3 & -l/6 \\ -l/6 & l/3 \end{bmatrix}$$

Q:- Formulate the flexibility matrix for the beam having the Co-ordinates as shown in the figure below.



To generate the first Column of flexibility matrix apply a Unit moment 1kN·m at Co-ordinate 1 as shown in figure.



$V_a + V_b = 0$

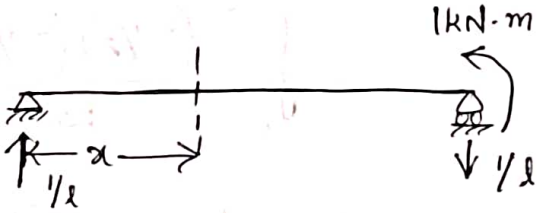
CW = ACW

$1 + V_a \times l = 0$

$V_a = -1/l$ and $V_b = 1/l$.

$m_1 = -\frac{1}{l} \times x + 1 = (1 - x/l)$

⇒ To generate 2nd Column



$\Sigma M_A = 0$ CW = ACW

$\Rightarrow -V_b \times l = 0$

$V_b = -1/l$ and $V_a = 1/l$

$M_2 = \frac{1}{l} \times x = \frac{x}{l}$

Step ② :- Flexibility Co-effecients.

$$\delta_{ij} = \int_0^l \frac{m_i m_j}{EI} \cdot dx.$$

$$\delta_{11} = \int_0^l \frac{m_1 \times m_1}{EI} dx = \int_0^l \frac{1}{EI} \left[(1 - l/x)(1 - l/x) \right] dx$$

$$= \frac{l}{3EI}$$

$$\delta_{21} = \int_0^l \frac{M_2 \cdot m_1}{EI} dx = \frac{1}{EI} \int_0^l \left(\frac{x}{l}\right)\left(1 - \frac{l}{x}\right) dx$$

(11)

$$= \frac{1}{EI}\int_0^l \left[\frac{x}{l} - \frac{x^2}{l^2}\right] dx$$

$$= \frac{1}{EI}\left[\frac{x^2}{2l} - \frac{x^3}{3l^2}\right]_0^l$$

$$\frac{1}{EI}\left[\frac{l^2}{2l} - \frac{l^3}{3l^2}\right] - 0$$

$$= \frac{1}{EI}\left[\frac{l}{2} - \frac{l}{3}\right]$$

$$= \frac{l}{6EI}$$

$\delta_{12} = \frac{l}{6EI}$ and

$$\delta_{22} = \frac{1}{EI}\int_0^l \left(\frac{x}{l}\right)\left(\frac{x}{l}\right) dx$$

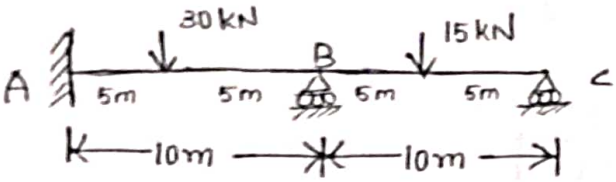
$$\frac{1}{EI}\int_0^l \left[\frac{x^2}{l}\right]^2 dx.$$

$$= \frac{1}{EI}\left[\frac{x^3}{3l^2}\right]_0^l$$

$$= \frac{1}{EI}\left[\frac{l^3}{3l^2}\right] = \frac{l}{3EI}.$$

$$[\delta] = \frac{1}{EI}\begin{bmatrix} l/3 & l/6 \\ l/6 & l/3 \end{bmatrix}$$

Q:- Analyse the Continuous beam by Flexibility method.



Sol:- Let the basic determination Structure we Obtained by realising the moments at A and B. This Can be done by Converting a fixed Support to a hinged Support at 'A' and by inserting internal hinge at B. So that Spans AB and BC behaves like two Simply Supported beams.

In flexibility method the Unknown moments or reactions may be found Out by Using the formula

$$[P] = [\delta]^{-1}\{[\Delta] - [\Delta_L]\}.$$

The degree of Static indeterminacy for beam.

$$D_S = r - 3 \Rightarrow 5 - 3 = 2$$

Based on above information Convert the fixed to a hinged Support.



$\Rightarrow D_S = 0$.

Note:-

1] 

$$[\delta] = \begin{bmatrix} l/3EI & l/6EI \\ l/6EI & l/3EI \end{bmatrix}$$

2] 

$$[\delta] = \begin{bmatrix} l/3EI & -l/6EI \\ -l/6EI & l/3EI \end{bmatrix}$$

Step ② :- Generation of flexibility matrix

To generate the first column of flexibility matrix. Apply a unit moment that is 1 kN-m at co-ordinate 1:

δ_{11} (displa / force) $= l/3EI = \frac{10}{3EI}$

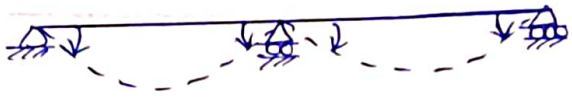
$\delta_{21} = l/6EI = 10/6EI = \frac{5}{3EI} = \delta_{12}$

$\delta_{22} = \frac{l}{3EI} + \frac{l}{3EI} = \frac{10}{3EI} + \frac{10}{3EI} = \frac{20}{3EI}$

$\therefore [\delta] = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} = \begin{bmatrix} 10/3EI & 5/3EI \\ 5/3EI & 20/3EI \end{bmatrix}$ Δ = Slope or displacements

Step ③ :- Displacements due to loads

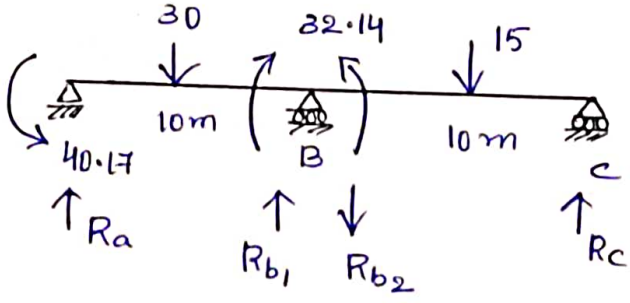
$\Delta_{1L} = \frac{wl^2}{16EI} = \frac{30\times10^2}{16EI} = \frac{187.5}{EI}$



$\Delta_{2L} = \frac{wl^2}{16EI} + \frac{wl^2}{16EI} = \frac{30\times10^2}{16EI} + \frac{15\times10^2}{16EI} = \frac{281.25}{EI}$

$$\begin{bmatrix} M_a \\ M_b \end{bmatrix} = EI \begin{bmatrix} 10/3 & 5/3 \\ 5/3 & 20/3 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 187.5/EI \\ 281.25/EI \end{bmatrix} \right\}$$

$$= \begin{bmatrix} -40.17 \\ -32.14 \end{bmatrix}$$



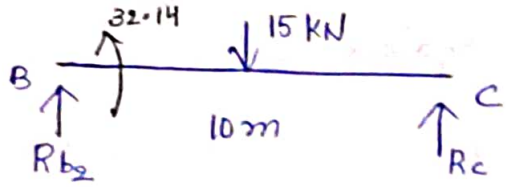
Step ④ :- Shear force and BMD.

$R_a + R_{b_1} = 30$ kN.

$\Sigma M_A = 0$ CW = ACW

$\Rightarrow 32.14 + 30\times5 = 40.17 + R_{b_1}\times10$

$= R_{b_1} = 14.19$ kN and $R_A = 30 - 14.19 = 15.8$ kN.



$R_{b_2} + R_c = 15\ kN$

$\Sigma M_B = 0$

CW = ACW.

$\Rightarrow R_{b_2} \times 10 = 32.14 + 15 \times 5$

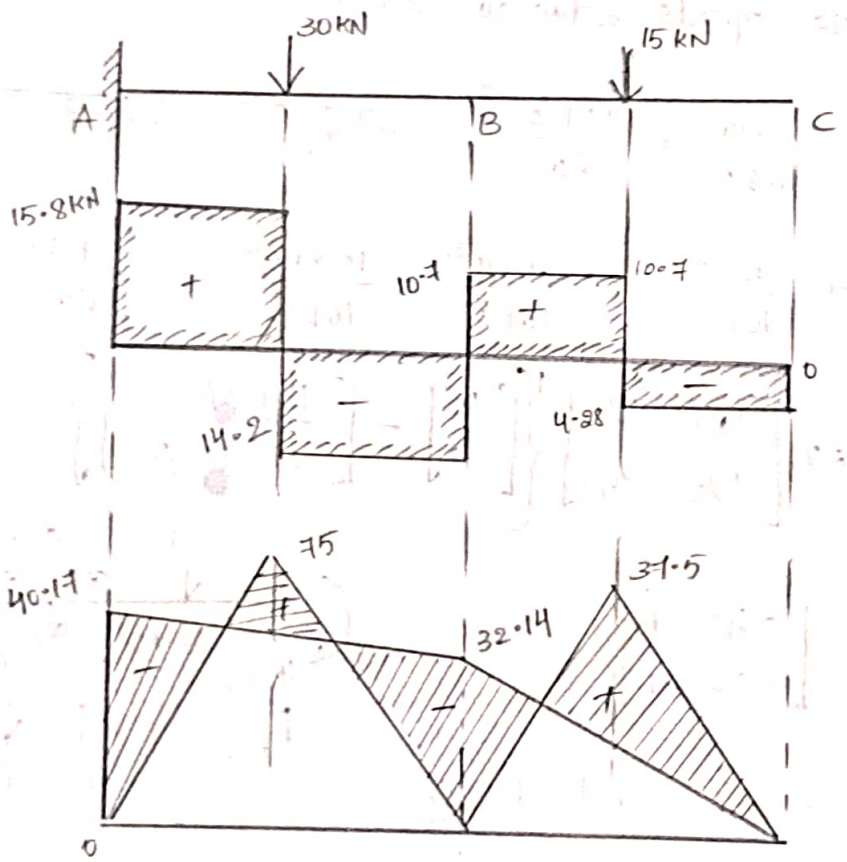
$R_{b_2} = 10.71\ kN$ and $R_c = 4.28\ kN$.

$R_b = R_{b_1} + R_{b_2} = 10.71 + 14.19 = 24.9\ kN$.

Free Bending moment:-

Span AB $= \frac{wl}{4} = \frac{30 \times 10}{4} = 75\ kN.m$.

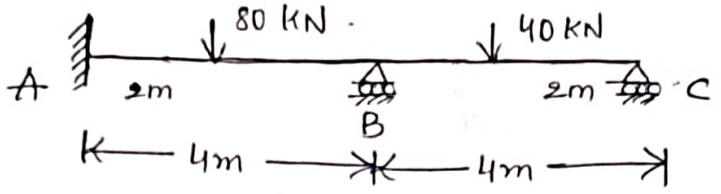
Span BC $= \frac{wl}{4} = \frac{15 \times 10}{4} = 37.5\ kN.m$.



$\delta_{12} = \frac{6EI}{}$ $= \frac{6EI}{}$ and $\delta_{22} = \frac{3EI}{}$ $\frac{3EI}{}$ $\frac{3EI}{}$

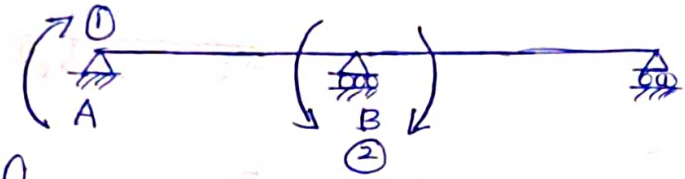
(1)

Q:- Analyze the Continuous beam Shown in figure, if the downward Settlement of Supports B and C are 10 mm and 5 mm respectively. Take $EI = 184 \times 10^{11}$ N·mm². Use flexibility matrix method.



Sol:- The degree of Static Indeterminacy $D_s = r - 3 \Rightarrow 5 - 3 = 2$

$\Rightarrow$ Convert the fixed Support to a hinged Support.



$D_s = 0$.

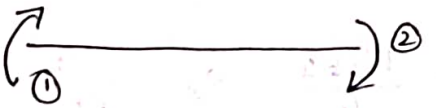
In flexibility method the Unknown moments or reactions may be found out by using the formula

$$[P] = [\delta]^{-1}\{[\Delta] - \Delta_L\}$$

Note:-

1] 

$$[\delta] = \begin{bmatrix} l/3EI & l/6EI \\ l/6EI & l/3EI \end{bmatrix}$$

2] 

$$[\delta] = \begin{bmatrix} l/3EI & -l/6EI \\ -l/6EI & l/3EI \end{bmatrix}.$$

Step ②:- Generation of flexibility matrix:

To generate the first Column of flexibility matrix. Apply a Unit moment that is 1 kN·m at Co-ordinate 1.

$$\delta_{11} = \frac{l}{3EI} = \frac{4}{3EI} \quad \text{and} \quad \delta_{21} = \frac{l}{6EI} = \frac{4}{6EI}$$

To generate the Second Column of flexibility matrix. Apply a Unit moment that is 1 kN·m at Co-ordinate 2:

$$\delta_{12} = \frac{l}{6EI} = \frac{4}{6EI} \quad \text{and} \quad \delta_{22} = \frac{l}{3EI} + \frac{l}{3EI} = \frac{4}{3EI} + \frac{4}{3EI} = \frac{8}{3EI},,$$

$$[\delta] = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} = \begin{bmatrix} 4/3 & 2/3 \\ 2/3 & 8/3 \end{bmatrix}$$

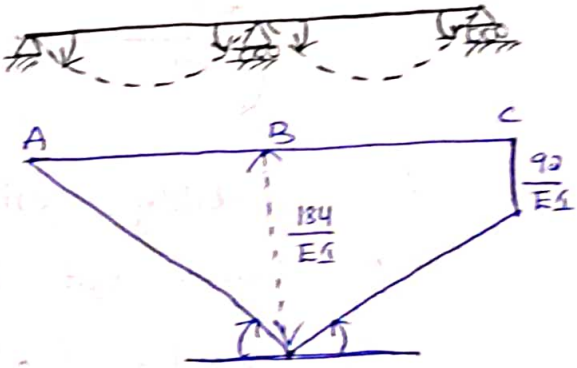
Step ③ :- Displacements due to loads :-

$$\Delta_{1L} = \frac{wl^2}{16EI} = \frac{80 \times 4^2}{16EI} = \frac{80}{EI}$$

$$\Delta_{2L} = \frac{wl^2}{16EI} + \frac{wl^2}{16EI} = \frac{80 \times 4^2}{16EI} + \frac{40 \times 4^2}{16EI} = \frac{120}{EI}.$$

Displacements due to Settlements :-

$$EI = 184 \times 10^{11}\ N \cdot mm^2$$
$$= 18400\ KN \cdot m^2$$



a) at Support 'B' Sinking by 10 mm.

$$10\ mm = \frac{184}{EI}.$$

b) at Supports 'c' Sinking by 5 mm. $= \frac{92}{EI}$

$$\Delta_{1L} = \frac{184/EI}{l} = \frac{184/EI}{4} = \frac{46}{EI}$$

$$\Delta_{2L} = \frac{-184/EI}{l} \Rightarrow \frac{-184/EI}{4} = \frac{92/EI}{4} \Rightarrow \frac{-69}{EI}$$

Total displacement due to loads and Settlements.

$$= [\Delta]_{loads} + [\Delta]_{Settle}$$

$$= \begin{bmatrix} 80/EI \\ 120/EI \end{bmatrix} + \begin{bmatrix} 46/EI \\ -69/EI \end{bmatrix}$$

$$\Delta_L = \begin{bmatrix} 126/EI \\ 51/EI \end{bmatrix}$$

Reactions in the beam is Calculated by.

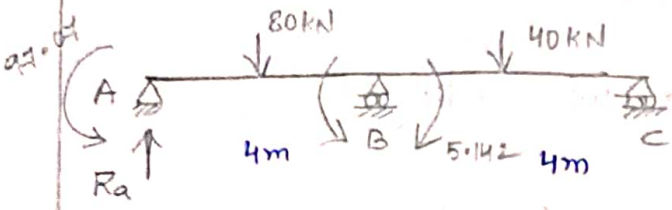
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$$[P] = [\delta]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$\begin{bmatrix} M_a \\ M_b \end{bmatrix} = EI \begin{bmatrix} 4/3 & 2/3 \\ 2/3 & 8/3 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{1}{EI} \begin{bmatrix} 126 \\ 51 \end{bmatrix} \right\}$$

$$\begin{bmatrix} M_a \\ M_b \end{bmatrix} = \begin{bmatrix} -97.07 \\ 5.142 \end{bmatrix}$$





Reactions:- $R_a + R_{b1} = 80$ kN.

CW = ACW

$\Sigma M_A = 0$ $\quad R_{b1} \times 4 + 5.142 + 97.07 = 80 \times 2$

$R_{b1} = 6.947$ kN.

$R_a = 73.053$ kN.

$R_{b2} + R_c = 40$ kN

$R_c \times 4 = 40 \times 2 + 5.142$

$R_c = 21.28$ kN

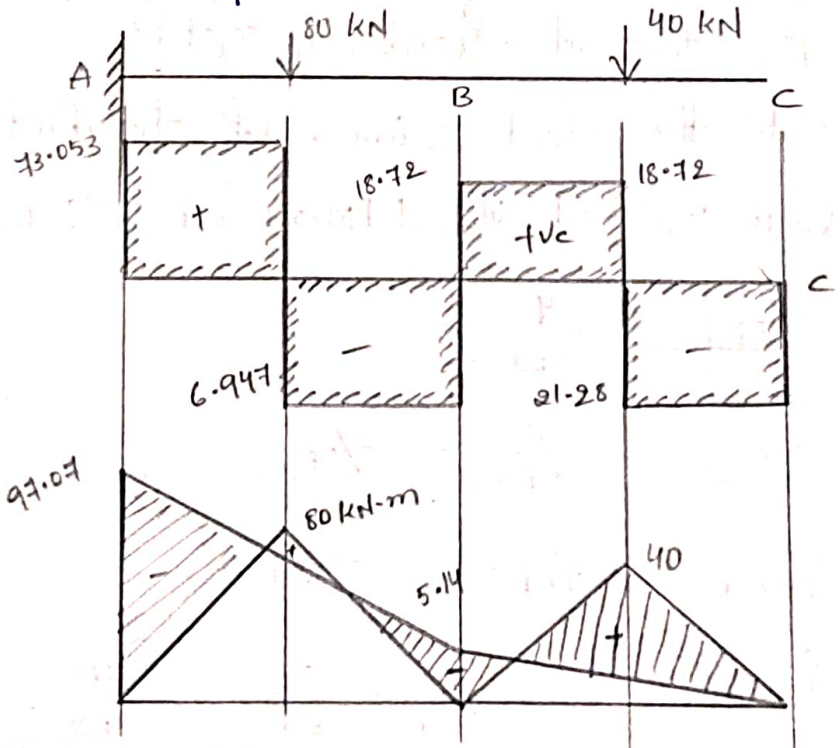
$R_{b2} = 18.72$ kN.

$R_b = 6.947 + 18.72 = 25.667$ kN.

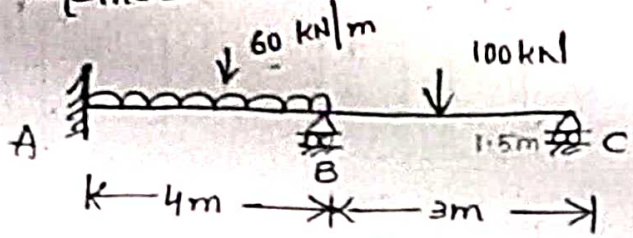
Free end Moments

Span AB $= \frac{wl}{4} = \frac{80 \times 4}{4} = 80$ kN-m.

Span Bc $= \frac{wl}{4} = \frac{40 \times 4}{4} = 40$ kN-m.

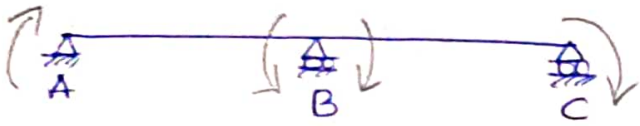


Q:- Analyze the Continuous beam Shown in figure below by flexibility method.



Sol:- The degree of Static Indeterminacy $D_s = 5-3 = 2$.

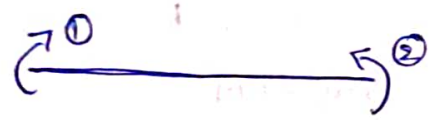
Convert the fixed Support into Hinged Support.



$D_s = 0$.

In flexibility method the Unknown moments or reactions may be found Out by Using the formula.

$$[P] = [\delta]^{-1}\{[\Delta] - [\Delta_L]\}.$$

Note:- 

$$[\delta] = \begin{bmatrix} l/3EI & l/6EI \\ l/6EI & l/3EI \end{bmatrix}$$

② 

$$[\delta] = \begin{bmatrix} l/3EI & -l/6EI \\ -l/6EI & l/3EI \end{bmatrix}.$$

Step ② :- ~~Displacements due to Loads~~

Generation of flexibility matrix.

To generate the first Column of flexibility matrix. Apply a Unit moment of 1 kN·m at Co-ordinate 1.

$$\delta_{11} = l/3EI = \frac{4}{3EI}.$$

$$\delta_{12} = l/6EI = \frac{4}{6EI} = 2/3EI.$$

$$\delta_{21} = l/6EI = 4/6EI = 2/3EI.$$

$$\delta_{22} = \frac{l}{3EI} + \frac{l}{3EI} = \frac{4}{3EI} + \frac{3}{3EI} = \frac{7}{3EI}$$

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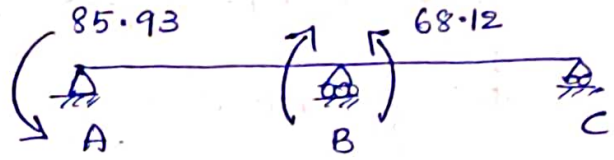
Step ③ :- Displacements due to loads.

$$\Delta_{1L} = \frac{wl^3}{24EI} = \frac{60\times4^3}{24EI} = \frac{160}{EI}$$

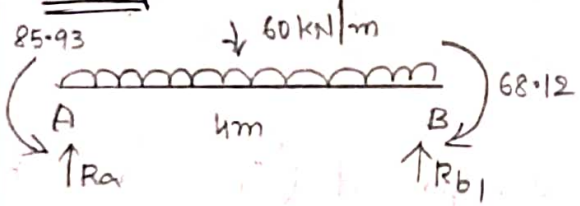
$$\Delta_{2L} = \frac{wl^3}{24EI} + \frac{wl^2}{96EI} = \frac{60\times4^3}{24EI} + \frac{100\times3^2}{16EI} = \frac{216.25}{EI}$$

$$\begin{bmatrix} M_a \\ M_b \end{bmatrix} = EI \begin{bmatrix} 4/3 & 2/3 \\ 2/3 & 7/3 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 160/EI \\ 216.25/EI \end{bmatrix} \right\}$$

$$= \begin{bmatrix} -85.93 \\ -68.12 \end{bmatrix}$$



Reactions :-

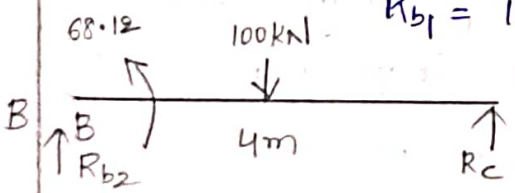


$$R_a + R_{b1} = 60\times4$$

$$R_a + R_{b1} = 240 \text{ kN}$$

$$\Sigma M_A = 0 \Rightarrow R_{b1}\times4 + 85.93 = 60\times4\times\frac{4}{2} + 68.12$$

$$R_{b1} = 115.5475 \text{ kN} \text{ and } R_a = 124.46 \text{ kN}.$$

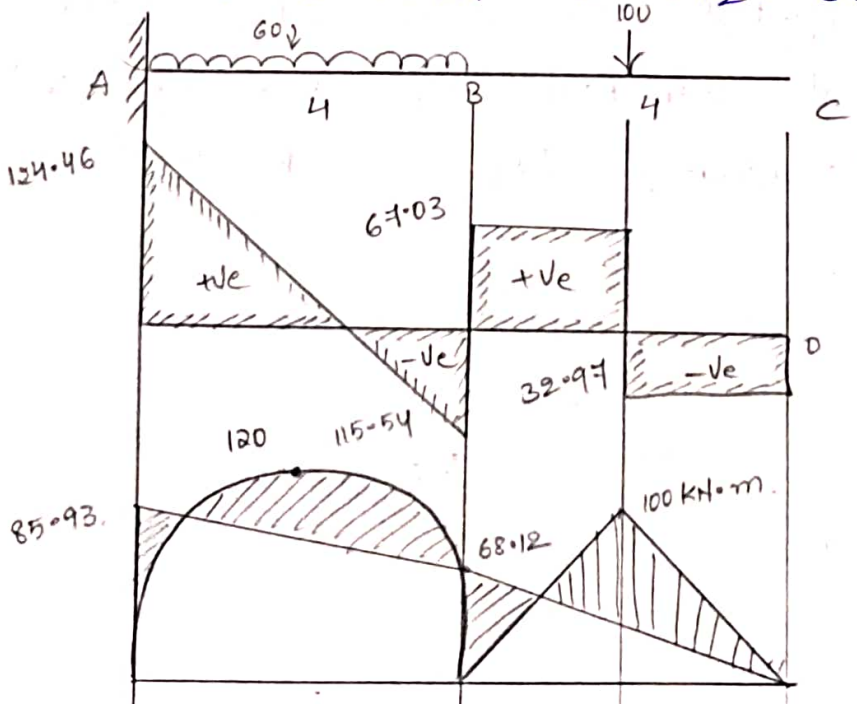


$$R_{b2} + R_c = 100 \text{ kN}.$$

$$\Sigma M_B = 0 \quad R_c\times4 + 68.12 = 100\times2$$

$$R_c = 32.97 \text{ kN} \text{ and } R_{b2} = 67.03$$

$$R_b = 182.577 \text{ kN}$$



$$\frac{wl^2}{8} = \frac{60\times4^2}{8} = 120.$$

$$\frac{wl}{4} = \frac{100\times4}{4} = 100$$