TEEGALA KRISHNA REDDY ENGINEERING COLLEGE

(UGC-Autonomous)

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DEPARTMENT OF CIVIL ENGINEERING
LECTURE NOTES

ACADEMIC YEAR: 2023-24

REGULATION: R20

YEAR/SEMESTER: III - I

SUBJECT: **STRUCTURALANALYSIS– II**

SUBJECT CODE: **20CE5PC04**

FACULTY NAME: P.CHAITANYA

SYLLABUS:

UNIT – V

Influence Lines – Definition of influence line for shear force and bending moment – load position for maximum shear force and maximum bending Moment at a section– Point loads, uniformly distributed load longer than the span, uniformly distributed load shorter than the span-Influence lines for forces in members of Prat tand Warren trusses–Equivalent uniformly distributed load–Focal length.

For Indeterminate Beams: Influence line diagram for shear force and bending moment for two span continuous beam with constant and different moments of inertia–influence line diagram for shear force and bending moment for propped cantilever beams.

Unit - V

Influence Lines for Indeterminate Beams

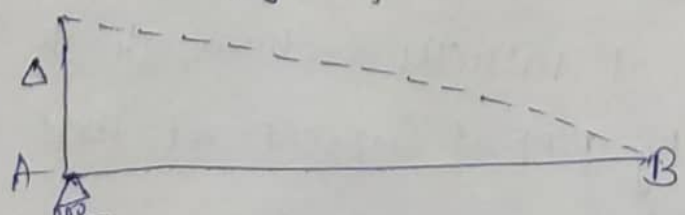
The concept of influence lines was introduced by E. Winkler in 1867. Prof. Muller-Breslau (1886) discovered an important principle by which influence lines for determinate as well as indeterminate structures can be found. This principle is based on Maxwell's theorem of reciprocal displacement.

Muller-Breslau principle

The Muller-Breslau principle may be stated as follows

statement: If an internal stress component (shearforce, Bending moment etc) or reaction component is allowed to act through a small distance thereby causing deformation of the structure, the curve of the deformed shape represents to some scale, the influence lines for that stress or the reaction component.

Explanation: To find the influence line diagram for reaction R_A in a simply supported beam, a small displacement



R_A shape of ILD for R_A in simply supported beam

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in the direction R_A is given. Its deflected shape shown by dotted line represents the influence lines for R_A to some s

similarly, if influence line diagram for moment M_A in a continuous beam is required, a small rotation is to be given at A. the deflected curve shown by dotted line represents the influence lines for moment M_A. The deformed shape of the frame shown in fig, shows the influence lines for horizontal thrust H_A.



Shape of ILD for M_A in continuous Beam

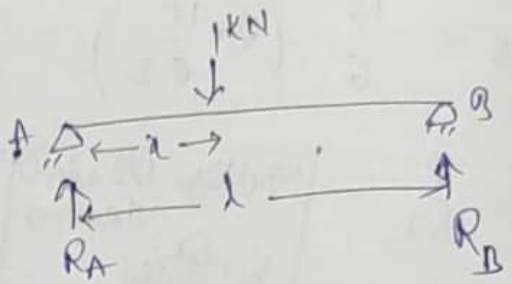
Stepwise procedure to draw ILD of indeterminate Beam

ILD for vertical reactions @ supports

1) Remove the ability to resist movement in the vertical direction at that support by removing the support.
2) Apply a unit force in the direction of support reaction
3) For this modified beam draw conjugate beam
4) calculate B.M's in conjugate beam which gives deflection in modified beam due to unit force
5) To get ILD ordinates at various sections, divide the BM of conjugate beam at that section by B.M at support at that section where ILD is to be plotted.
6) Draw influence line diagram

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ILD for Reaction



$\Sigma M_B = 0$

$R_A \times l = 1(l-x)$

$R_A = l - x/l = 1 - x/l$

$R_A + R_B = 1$

$1 - x/l + R_B = 1$

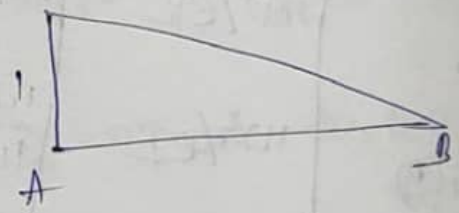
$R_B = x/l$

ILD for R_A :

$R_A = 1 - x/l$, at A

at $x = 0$, $R_A = 1$

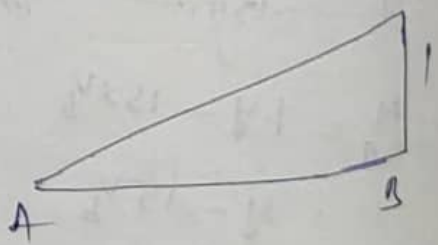
at B, at $x = l$, $R_A = 0$



ILD for R_B :

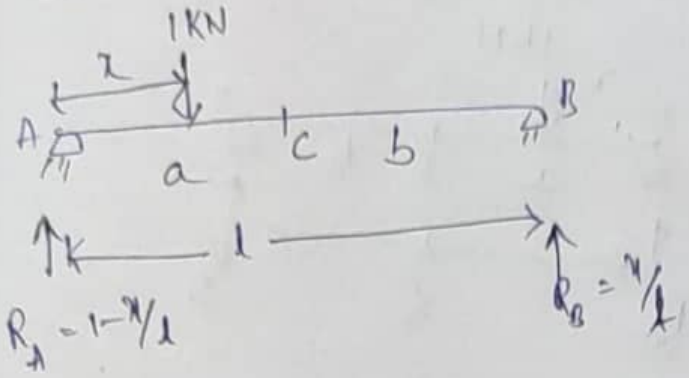
$R_B = x/l$, at A, $x = 0$, $R_B = 0$

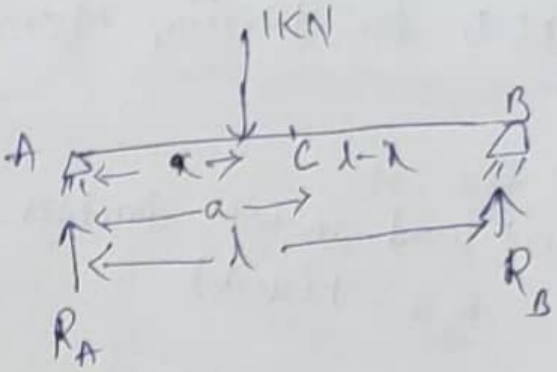
at B, $x = l$, $R_B = 1$



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ILD for shear force





shear force @ C :

span AC (left portion)

" CB (Right ")

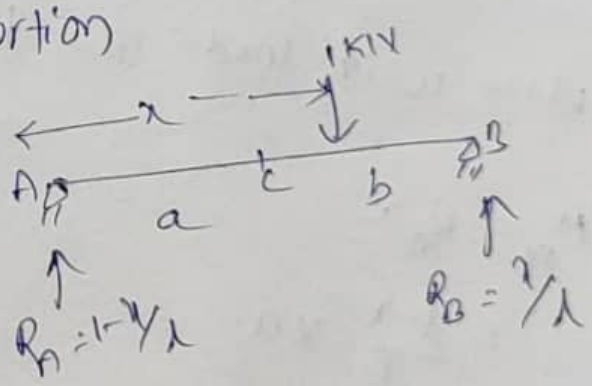
→ when unit load is on left portion

$$SF_C = R_A - 1 = 1 - x/l - 1 = -x/l$$

if $x = a$

$$\boxed{S.F_C = -a/l}$$

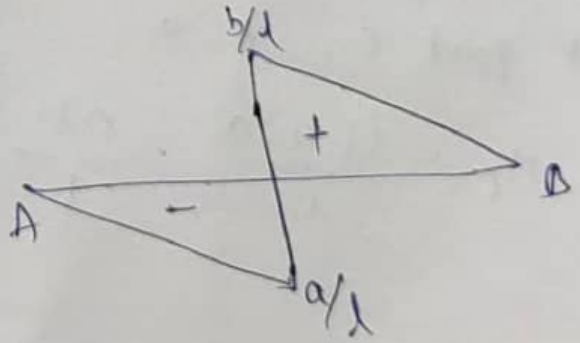
→ when unit load is on Right portion



$$S.F_C = 1 - x/l$$

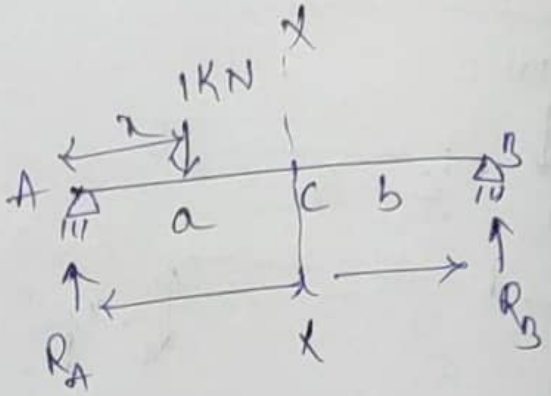
if $x = a$, $S.F_C = \frac{l-a}{l}$

$$\boxed{S.F_C = b/l}$$



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ILD for Bending Moment



ILD for M_C

i) When unit load in left portion

$$M_C = R_A a - 1(a-x)$$

$$= \left(\frac{l-x}{l}\right)a - a + x$$

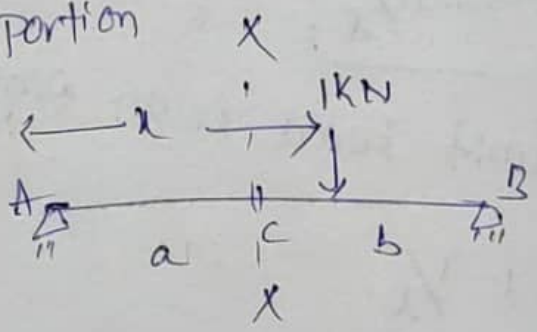
$$= \frac{al - ax - al + xl}{l}$$

$$M_C = \frac{xl - ax}{l}$$

if $x = a$, $M_C = \frac{al - a^2}{l} = \frac{a(l-a)}{l} = \frac{ab}{l}$

$$\boxed{M_C = \frac{ab}{l}}$$

ii) When unit load in Right portion

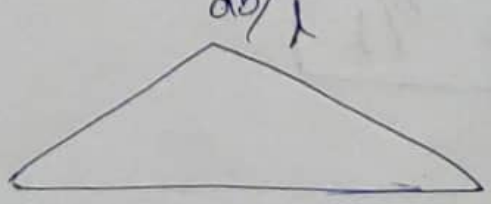


$$M_{XX} = R_A a$$

$$= \frac{l-x}{l} \times a$$

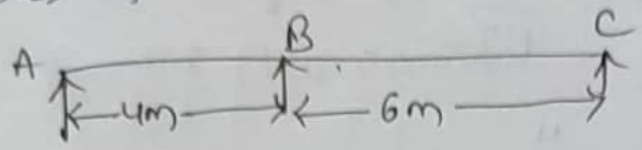
at point C, $x = a$

$$M_C = \frac{(l-a)a}{l} = \frac{ab}{l}$$

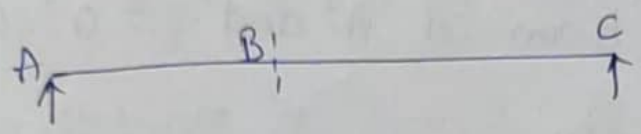


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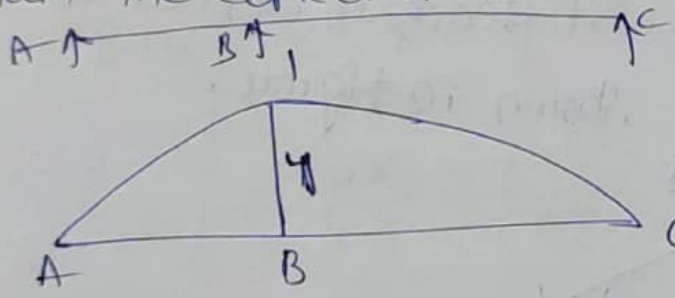
8] Draw influence line diagram for reaction at support 'B' for continuous beam ABC having span AB = 4m & BC = 6m. Plot ordinates of ILD at 1m interval.



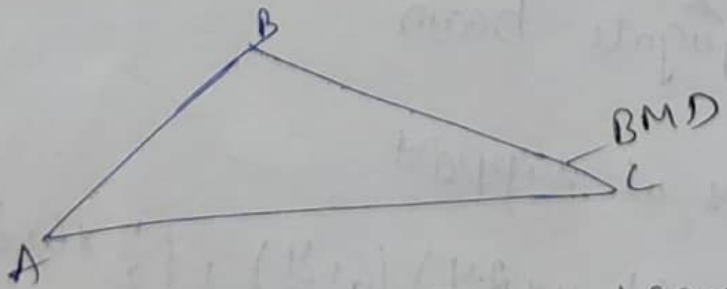
Sol:



- Remove the ability of the beam to resist the function
- Give unit displacement in the direction of the action (R_B)
- obtain the deflection curve due to unit displacement.



- the resultant deflection curve due to unit displacement itself is the ILD for the function at corresponding point.



To draw conjugate beam for beam given in figure due to simply support BM @ A & C = 0

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BM at $B = \frac{Wab}{l} = \frac{1 \times 4 \times 6}{10} = 2.4$

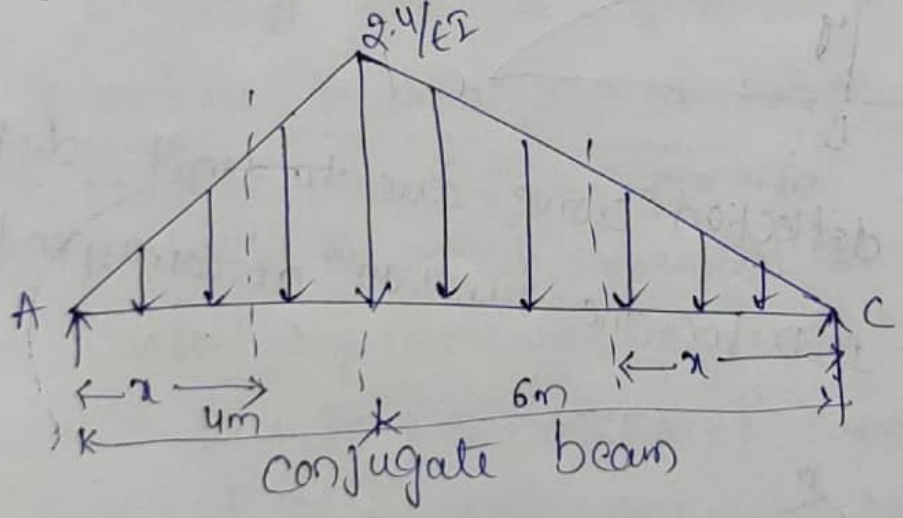
load on Conjugate Beam = $\frac{M}{EI}$ of original beam

Hence load on C.B = "

So load on conjugate beam at A and C = 0 and $B = \frac{2.4}{EI}$. Load varies linearly.

Modify the supports for conjugate beam. Simple suppo at A and C remain as it is. there is no support at B since it is removed and unit load is applied at its place

Conjugate beam is shown in figure



Calculate reactions at support

$\Sigma M_C = 0, \quad R_A \times 10 = \left(\frac{1}{2} \times 4 \times \frac{2.4}{EI}\right)\left(6 + \frac{4}{3}\right) + \left(\frac{1}{2} \times 6 \times \frac{2.4}{EI}\right) \times \frac{2 \times 6}{3}$

$R_A = \frac{6.4}{EI}$ (upwards)

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$\Sigma V = 0$

$$R_A + R_C = \frac{1}{2} \times \frac{2.4}{EI} \times 10$$

$$R_C = \frac{5.6}{EI} \text{ (upwards)}$$

General eqn of B.M of conjugate beam for portion AB

At any distance x from A $\frac{y}{x} = \frac{2.4/EI}{4}$

$$y = \frac{0.6x}{EI}$$

$$B.M_x = \frac{6.4}{EI} x - \frac{1}{2} x \times \frac{0.6}{EI} x \times \frac{x}{3}$$

$$= \frac{6.4x}{EI} - \frac{0.1x^3}{EI}$$

General eqn of BM of CB for portion CB at any distance x from C, $\frac{y}{x} = \frac{2.4/EI}{6}$

$$y = \frac{0.4x}{EI}$$

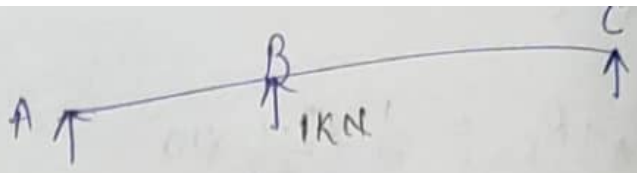
$$BM_x = \frac{5.6}{EI} x - \frac{1}{2} \times x \times \frac{0.4}{EI} x \times \frac{x}{3}$$

$$= \frac{5.6x}{EI} - \frac{0.4x^3}{6EI}$$

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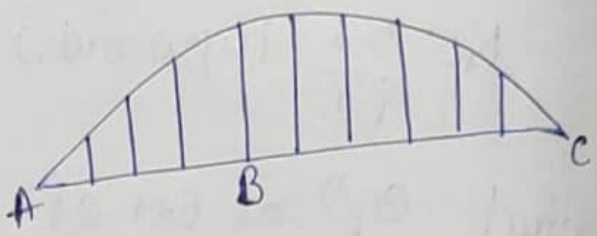
For AB Portion

$$BM_x = \frac{6.4}{EI}x - \frac{0.1}{EI}x^3$$



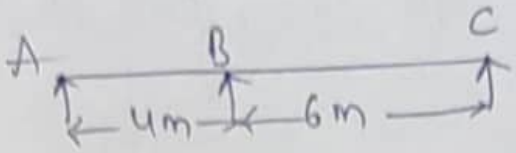
For CB portion

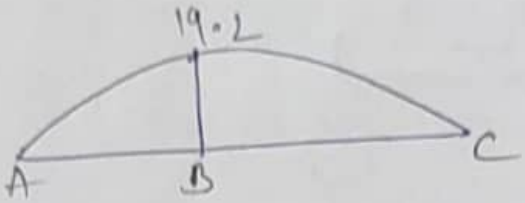
$$BM_x = \frac{5.6x}{EI} - \frac{0.4}{6EI}x^3$$

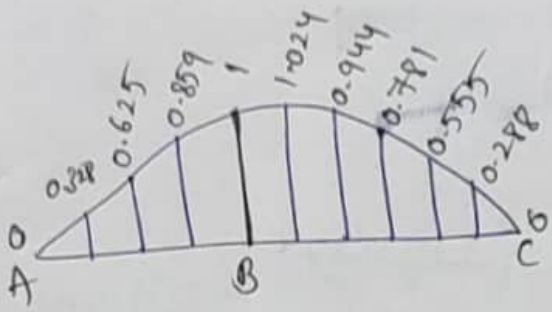


Section at	Equation	Distance from origin (x)	BM ordinate	ILD ordinate = BM ord / 19.2/EI
A	eqn for Portion AB (origin A)	0	0	0
1		1	6.3/EI	0.328
2		2	12/EI	0.625
3		3	16.5/EI	0.859
4		4	19.2/EI	1.0
5	eqn for Portion CB (origin C)	5	19.67/EI	1.024
6		4	18.133/EI	0.944
7		3	15/EI	0.7813
8		2	10.66/EI	0.553
9		1	5.53/EI	0.2886
C		0	0	0

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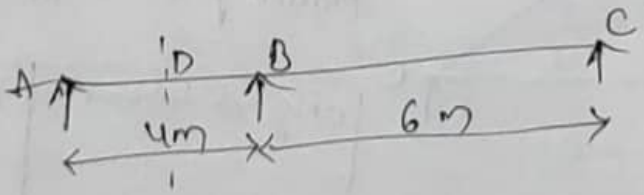




Influence line diagram

Influence line for BMD

Q) Draw influence line diagram for BM at section D, at midspan point of span AB of continuous beam ABC. span AB = 4m and BC = 6m. Find ordinates at 1m interval

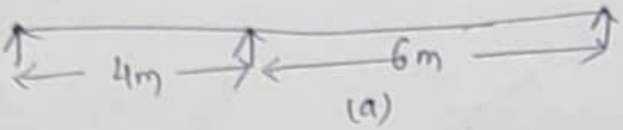


Sol:- To draw ILD for BM at D

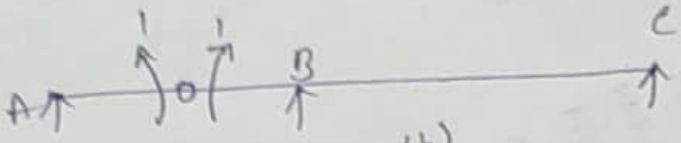
- Remove the ability to resist moment at D by introducing hinge as shown in fig(b)
- Apply unit moment @ D

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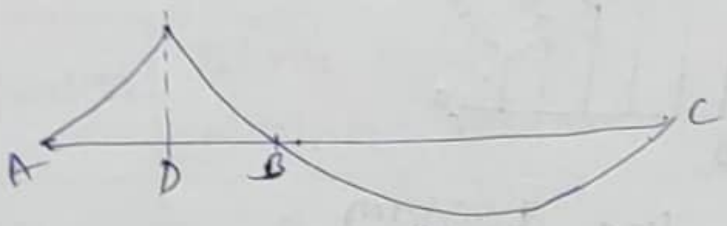
- Because of this unit moment the beam will deflect as shown in fig (c)



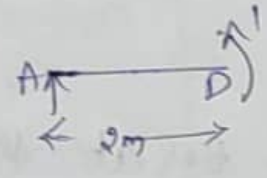
(a)



(b)



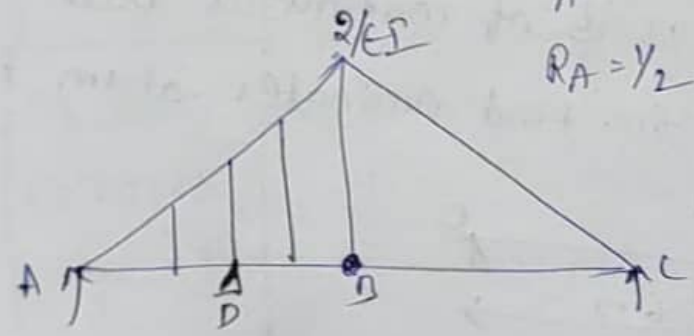
(c)



B.M @ D = 0

$R_A \times 2 = 1$

$R_A = 1/2$



Conjugate Beam

Modify the supports of conjugate beam

original beam	Conjugate beam
Roller end	Roller end
Hinge end	Hinge end
Fixed end	Free end
Free end	Fixed end
Interid support	Interid hinged
Interid hinged	Interid support

Σ B.M @ B = 0

$M_B = R_A \times 4 = 1/2 \times 4 = 2$ KN.m

B/M load on conjugate beam = $\frac{M}{EI}$ of original beam

$= \frac{2}{EI}$

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first Calculate Reaction at supports of conjugate beam

Consider portion BC

Since B is internal hinge, B.M @ B = 0

$$-R_C \times 6 + \frac{1}{2} \times \frac{2}{EI} \times 6 \times \frac{6}{3} = 0$$

$$R_C = 2/EI \text{ (downward)}$$

$\Sigma M_A = 0$,

$$R_C \times 10 + R_D \times 2 = \frac{1}{2} \times 4 \times \frac{2}{EI}\left(2 \times \frac{4}{3}\right) + \frac{1}{2} \times 6 \times \frac{2}{EI}\left(4 + \frac{6}{3}\right)$$

$$\frac{2}{EI} \times 10 + 2R_D = \frac{10.6}{EI} + \frac{36}{EI} = \frac{46.6}{EI}$$

$$2R_D = \frac{26.66}{EI}$$

$$R_D = \frac{13.33}{EI}$$

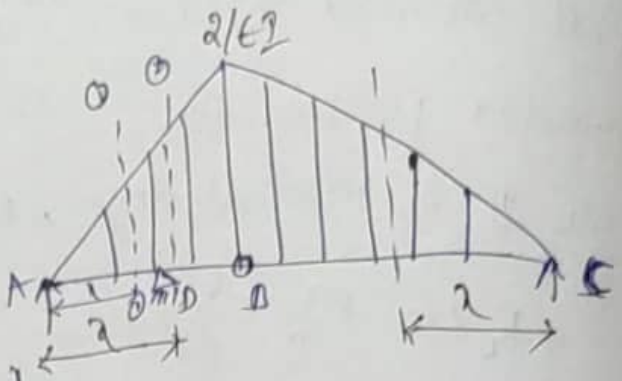
$\Sigma V = 0$, $R_A + R_D + R_C = \frac{1}{2} \times 10 \times \frac{2}{EI}$

$$R_A + \frac{13.33}{EI} + \frac{2}{EI} = \frac{10}{EI}$$

$$R_A = \frac{5.33}{EI}$$

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→ At any distance 'x' from A, sect ① a portion AD



$$\frac{y}{x} = \frac{2/EI}{4}$$

$$y = \frac{2x}{4EI}$$

$$BM_x = \frac{5.33}{EI}x + \frac{1}{2} \times x \times \frac{2x}{4EI} \times \frac{x}{3}$$

$$BM_x = \frac{5.33x}{EI} + \frac{x^3}{12EI} \quad \text{—— ①}$$

→ At any distance 'x' from A, sect ② a portion DB

$$\frac{y}{x} = \frac{2/EI}{4}$$

$$y = \frac{2x}{4EI} = \frac{x}{2EI}$$

$$BM_x = \frac{5.33x}{EI} + \frac{x^3}{12EI} - \frac{13.33}{EI}(x-2) \quad \text{—— ②}$$

→ At any distance 'x' from C, $\frac{y}{x} = \frac{2/EI}{6}$ (Portion CB)

$$y = \frac{x}{3EI}$$

$$BM_x = -\frac{2}{EI}x + \frac{1}{2} \times x \times x \times \frac{x}{3EI} \times \frac{x}{3}$$

$$= \frac{-2x}{EI} + \frac{x^3}{18EI} \quad \text{—— ③}$$

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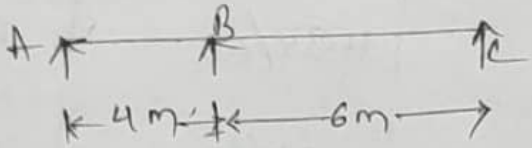
Section at	Equation	Distance from origin (x)	BM ordinate	ILD ordinate
A	eqⁿ for Portion AD (origin A) eqⁿ ①	0	0	0
1		1	5.41/EI	0.406
2		2	11.33/EI	0.85
3	Portion DB (origin A) eqⁿ ②	3	4.91/EI	0.368
4		4	0	0
5	eqⁿ for Portion CB (origin C) eqⁿ ③	5	-3.05/EI	-0.229
6		4	-4.44/EI	-0.33
7		3	-4.5/EI	-0.337
8		2	-3.55/EI	-0.267
9		1	-1.94/EI	-0.145
C		0		0





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Q) Draw influence line diagram for B.M at section D, at mid point of span AB of continuous beam ABC. Span AB = 4m, and BC = 6m. Find ordinates at midpoint of Span AB & BC



Ans:- Calculate reactions at A, D, C

$$R_A = 5.33/EI$$

$$R_D = 13.33/EI$$

$$R_C = 2/EI$$

To calculate ILD ordinate at D, first calculate B.M at D of conjugate beam and then divide it by R_D. Since change in shear force in conjugate beam corresponds to rotation in original beam.

$$M_D = \frac{5.33}{EI} \times 2 + \frac{1}{2} \times 2 \times \frac{1}{EI} \left(\frac{2}{3}\right)$$

$$M_D = 11.332/EI$$

Similarly to calculate ILD ordinate at midpoint of span BC, say at 'E'.

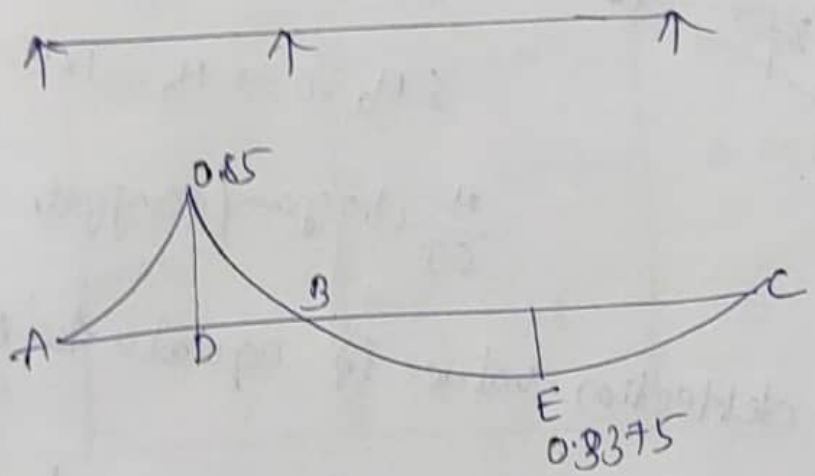
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$$M_E = -\frac{2}{EI} \times 3 + \frac{1}{2} \times 3 \times \frac{1}{EI} \left(\frac{3}{3}\right)$$

$$= -4.5/EI$$

ILD ordinate at E $= \dfrac{-4.5/EI}{13.33/EI} = -0.3375$

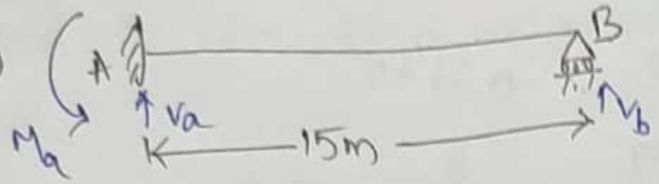
ILD ordinate at D $= \dfrac{11.332/EI}{13.33/EI} = 0.85$



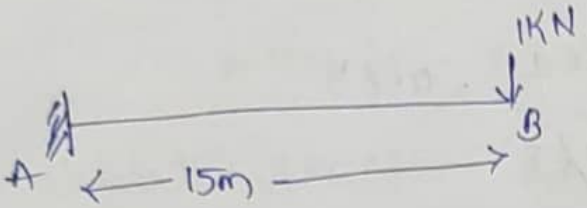
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ILD for propped cantilever Beam

Q)



Sol:



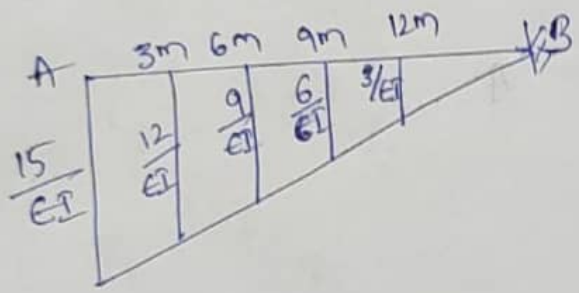
Muller-Breslau theory

Let us Redundant $R_B = V_b$

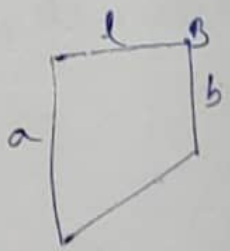
$\Sigma V = 0 \Rightarrow R_A = 1$

$\Sigma M_a = 0 \Rightarrow M_a = 1 \times 15 = 15$

$\frac{M}{EI}$ diagram / conjugate beam



In actual beam deflection value is equal to B.M in conjugate beam.



$M_B = \frac{l^2}{6}(2a+b)$

$\delta_A = \delta_{0m} = 0$

$\delta_{3m} = \frac{3^2}{6}\left(2 \times \frac{15}{EI} + \frac{12}{EI}\right) = \frac{63}{EI}$

$\delta_{6m} = \frac{6^2}{6}\left(2 \times \frac{15}{EI} + \frac{9}{EI}\right) = \frac{234}{EI}$

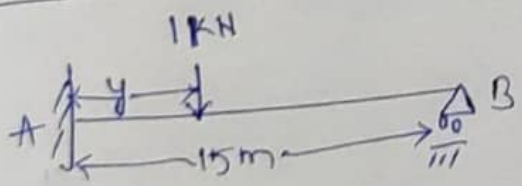
$\delta_{9m} = \frac{9^2}{6}\left(2 \times \frac{15}{EI} + \frac{6}{EI}\right) = \frac{486}{EI}$

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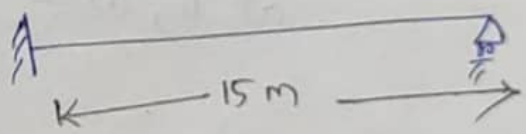
$$\delta_{12m} = \frac{12^2}{6}\left(2\times\frac{15}{EI} + \frac{3}{EI}\right) = 792/EI$$

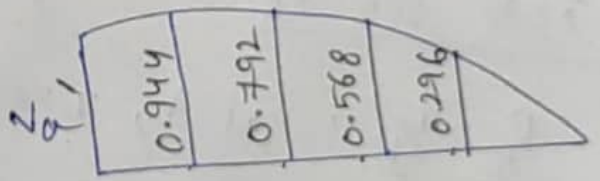
$$\delta_{15m} = \delta_B = \frac{15^2}{6}\left(2\times\frac{15}{EI}\right) = \frac{1125}{EI}$$

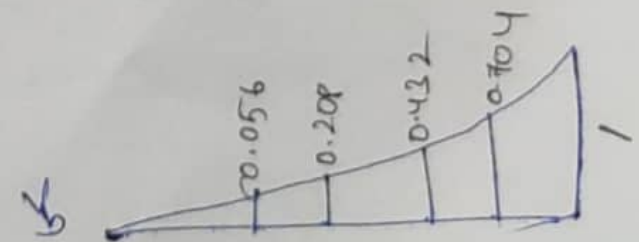
Location from support A	δ_2	Muller Breslau theorem $V_b = \delta_2/\delta_B$	$V_a = 1 - V_b$	$M_a = y - 15V_b$
0m (A)	0	0	1−0 = 1	$M_a = 0 - 15\times0 = 0$
3m	63/EI	$\frac{63/EI}{1125/EI} = 0.056$	1−0.056 = 0.944	$M_3 = 3 - 15\times0.056 = 2.16$
6m	234/EI	234/1125 = 0.208	0.792	$M_6 = 6 - 15(0.208) = 2.88$
9m	486/EI	486/1125 = 0.432	0.568	$M_9 = 9 - 15(0.432) = 2.52$
12m	792/EI	792/1125 = 0.704	0.296	$M_{12} = 12 - 15(0.704) = 1.44$
15m (B)	1125/EI	$\frac{1125}{1125} = 1$	0	$M_{15} = 15 - 15(1) = 0$

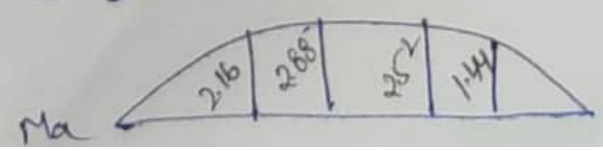


$$M_a = 1\cdot y - 15\times V_b$$
$$= y - 15V_b$$









ILD Diagrams

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This principle gives an excellent experimental approach to determine the influence lines for indeterminate structures. Using wire models, one can find influence lines by giving small displacements and marking deformed shapes. Another experimental technique is by using Begg's deformator. Begg's deformator is a set of equipments developed for introducing the required displacement in persplex models. Using microscope, the displacements are measured in the perspex model and influence lines are determined.

In this text, the analytical method is explained for finding influence lines for various stress and reaction components in two span continuous beams. The analytical method consists of releasing restraint for the stress/reaction component for which influence line diagram is required and finding displacement at various points due to unit force applied. For finding displacement at various points, any one method may be employed. Conjugate beam method is more convenient to use and is explained with few examples.

Example 5.1 Compute the ordinates of influence lines for reaction R_A for the beam shown in Figure 5.6(*a*) at 1 m interval and draw the influence line diagram. The moment of inertia is constant throughout.

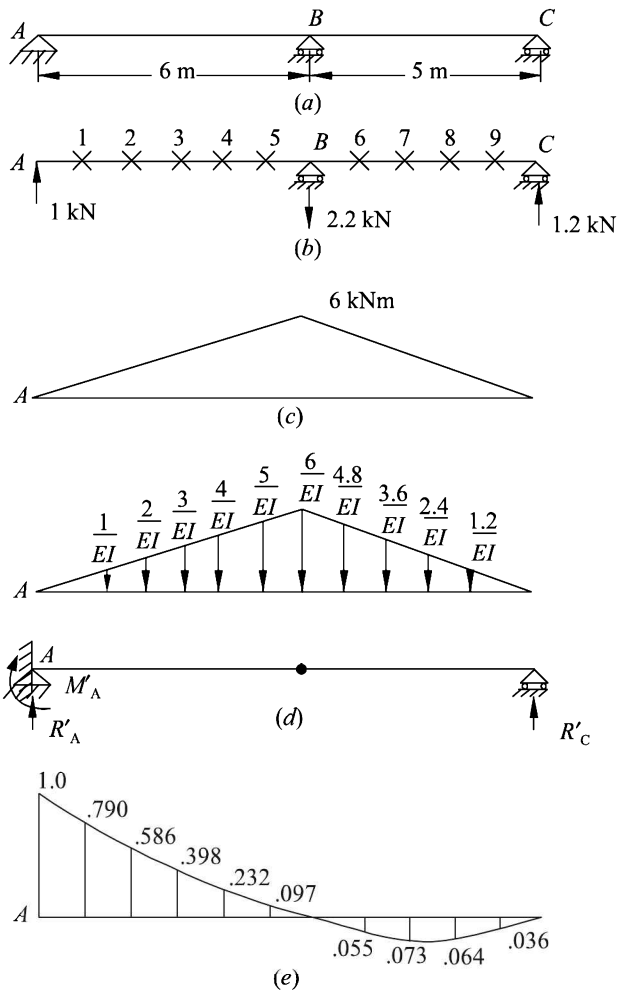


Figure 5.6(*a*): Given beam. **(*b*)** Unit load at *A* in released beam. **(*c*)** Bending moment diagram for released beam. **(*d*)** Conjugate beam. **(*e*)** ILD for R_A.

Solution Restraint in the direction of R_A is removed and a unit load in the direction of R_A is applied as shown in Figure 5.6(*b*). According to Müller-Breslau principle, the deflected shape of this beam is the influence line diagram for R_A. The bending moment diagram for the released beam is as shown in Figure 5.6(*c*). To find the deflections in this released structure, consider its conjugate beam with $\left(\frac{M}{EI}\right)$ diagram as load (see Figure 5.6(*d*)).

In the conjugate beam,

$$\Sigma M_B = 0, \text{ gives}$$

$$R'_C \times 5 = \frac{1}{2} \times 5 \times \frac{6}{EI} \times \frac{5}{3}$$

$$R'_C = \frac{5}{EI}$$

$$\Sigma V = 0, \text{ gives}$$

$$R'_A + R'_C = \frac{1}{2} \times 11 \times \frac{6}{EI} = \frac{33}{EI}$$

$$R'_A = \frac{33}{EI} - \frac{5}{EI} = \frac{28}{EI}$$

$$\Sigma M_A = 0, \text{ gives}$$

$$\therefore \quad M'_A + \frac{5}{EI} \times 11 - \frac{1}{2} \times 11 \times \frac{6}{EI}\left(\frac{11+6}{3}\right) = 0$$

Note: *C.G. of triangle from end A* $= \left(\frac{L+a}{3}\right)$ *with usual notations.*

$$\therefore \quad M'_A = \frac{132}{EI}$$

Therefore, support reactions are as shown in Figure 5.6(*e*).

Taking moment causing tension at the bottom (sagging) as positive and the one causing tension at the top (hogging) as negative, we get,

$$M_A = -\frac{132}{EI}$$

$$M_1 = -\frac{132}{EI} + \frac{28}{EI} \times 1 - \frac{1}{2} \times 1 \times \frac{1}{EI} \times \frac{1}{3} = -\frac{104.33}{EI}$$

$$M_2 = -\frac{132}{EI} + \frac{28}{EI} \times 2 - \frac{1}{2} \times 2 \times \frac{2}{EI} \times \frac{2}{3} = -\frac{77.33}{EI}$$

$$M_3 = -\frac{132}{EI} + \frac{28}{EI} \times 3 - \frac{1}{2} \times 3 \times \frac{3}{EI} \times 1 = -\frac{52.5}{EI}$$

$$M_4 = -\frac{132}{EI} + \frac{28}{EI} \times 4 - \frac{1}{2} \times 4 \times \frac{4}{EI} \times \frac{4}{3} = -\frac{30.67}{EI}$$

$$M_5 = -\frac{132}{EI} + \frac{28}{EI} \times 5 - \frac{1}{2} \times 5 \times \frac{5}{EI} \times \frac{5}{3} = -\frac{12.83}{EI}$$

$$M_B = 0$$

For portion BC, moments are calculated from the right side end.

$$M_C = 0$$

$$M_9 = \frac{5}{EI} \times 1 - \frac{1}{2} \times 1 \times \frac{1.2}{EI} \times \frac{1}{3} = \frac{4.8}{EI}$$

$$M_8 = \frac{5}{EI} \times 2 - \frac{1}{2} \times 2 \times \frac{2.4}{EI} \times \frac{2}{3} = \frac{8.4}{EI}$$

$$M_7 = \frac{5}{EI} \times 3 - \frac{1}{2} \times 3 \times \frac{3.6}{EI} \times 1 = \frac{9.6}{EI}$$

$$M_6 = \frac{5}{EI} \times 4 - \frac{1}{2} \times 4 \times \frac{4.8}{EI} \times \frac{4}{3} = \frac{7.2}{EI}$$

The above moments in the conjugate beam represent deflections in the released beam; and the deflections in the released beam represent the influence line diagram in the given beam.

Now, Influence line diagram (ILD) ordinate at A

= Deflection at A in released beam

= Moment in conjugate beam at $A = \frac{132}{EI}$

But, we known ILD ordinate at A for R_A is unity. Hence, dividing moment values in conjugate beam by $\frac{132}{EI}$, we get ILD for R_A in the given beam ILD for R_A, thus obtained, is shown in Figure 5.6(*e*).

Example 5.2 Find the influence line diagram for reaction at B in the continuous beam shown in Figure 5.7(*a*). Take EI as constant throughout.

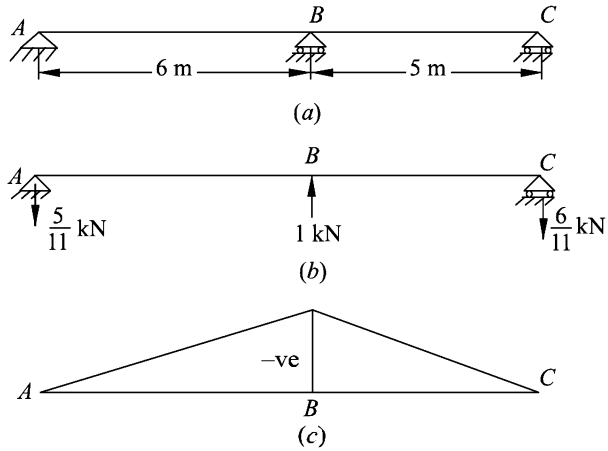


Figure 5.7(*a*): Continuous beam. **(*b*)** Released beam with unit load. **(*c*)** Bending moment diagram of released beam.

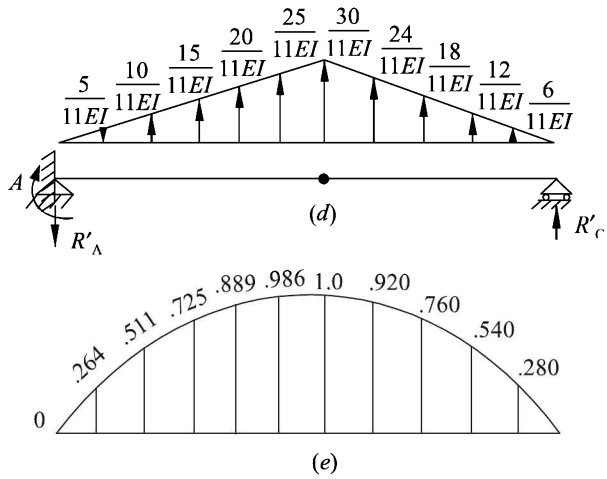


Figure 5.7(*d*): Conjugate beam. **(e)** ILD for R_B.

Solution Remove restraint in the direction of R_B and apply unit force as shown in Figure 5.7(*b*). According to Müller-Breslau principle, deflected shape of this beam represents the influence line for R_B in the given continuous beam.

Bending moment diagram for the released beam is as shown in Figure 5.7(*c*). Figure 5.7(*d*) shows the conjugate beam and loading on it for the released beam. In conjugate beam.

$\Sigma M_A = 0$, gives

$$R'_C \times 11 - \frac{1}{2} \times 11 \times \frac{30}{11EI}\left[\frac{11+6}{3}\right] = 0$$

$$R'_C = \frac{7.727}{EI} \downarrow$$

$$R'_A = \frac{1}{2} \times 11 \times \frac{30}{11EI} - \frac{7.727}{EI} = \frac{7.273}{EI} \uparrow$$

$$M_A = 0$$

$$M_1 = -\frac{7.273}{EI} \times 1 + \frac{1}{2} \times 1 \times \frac{5}{11EI} \times \frac{1}{3} = -\frac{7.197}{EI}$$

$$M_2 = -\frac{7.273}{EI} \times 2 + \frac{1}{2} \times 2 \times \frac{10}{11EI} \times \frac{2}{3} = -\frac{13.940}{EI}$$

$$M_3 = -\frac{7.273}{EI} \times 3 + \frac{1}{2} \times 3 \times \frac{15}{11EI} \times 1 = -\frac{19.774}{EI}$$

$$M_4 = -\frac{7.273}{EI} \times 4 + \frac{1}{2} \times 4 \times \frac{20}{11EI} \times \frac{4}{3} = -\frac{24.244}{EI}$$

$$M_5 = -\frac{7.273}{EI}\times 5+\frac{1}{2}\times 5\times\frac{25}{11\,EI}\times\frac{5}{3}=-\frac{26.895}{EI}$$

$$M_B = -\frac{7.273}{EI}\times 6+\frac{1}{2}\times 6\times\frac{30}{11\,EI}\times 2=-\frac{27.274}{EI}$$

For portion BC, moments are calculated from end C

$$M_C = 0$$

$$M_9 = -\frac{7.727}{EI}\times 1+\frac{1}{2}\times 1\times\frac{6}{11\,EI}\times\frac{1}{3}=-\frac{7.636}{EI}$$

$$M_8 = -\frac{7.727}{EI}\times 2+\frac{1}{2}\times 2\times\frac{12}{11\,EI}\times\frac{2}{3}=-\frac{14.727}{EI}$$

$$M_7 = -\frac{7.727}{EI}\times 3+\frac{1}{2}\times 3\times\frac{18}{11\,EI}\times 1=-\frac{20.726}{EI}$$

$$M_6 = -\frac{7.727}{EI}\times 4+\frac{1}{2}\times 4\times\frac{24}{11\,EI}\times\frac{4}{3}=-\frac{25.090}{EI}$$

Influence line diagram (ILD) ordinate for R_B at B in a given beam

= Deflection at B in released beam

= Moment in conjugate beam at $B = -\dfrac{27.274}{EI}$

But ILD ordinate for R_B is unity. Hence, by dividing all the moments in the conjugate beam by $-\dfrac{27.274}{EI}$, we get ILD ordinates for R_B at various points. These values are shown in Figure 5.5(e) and the ILD is drawn.

Example 5.3 Draw the influence line diagram for moment at B in the continuous beam shown in Figure 5.8(a) after calculating ordinates at 2 m intervals. Assume flexural rigidity is constant throughout.

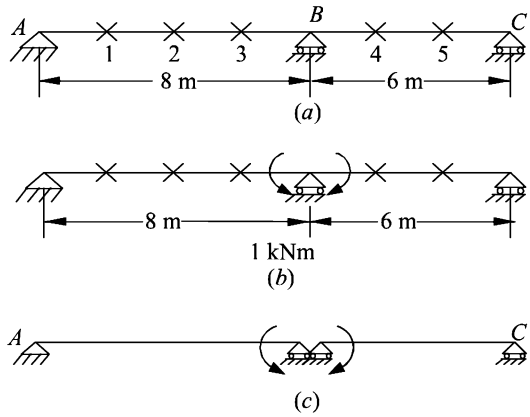


Figure 5.8(*a*): Continuous beam. **(*b*)** Released beam with unit moment at B. **(c)** Equivalent of released beam.

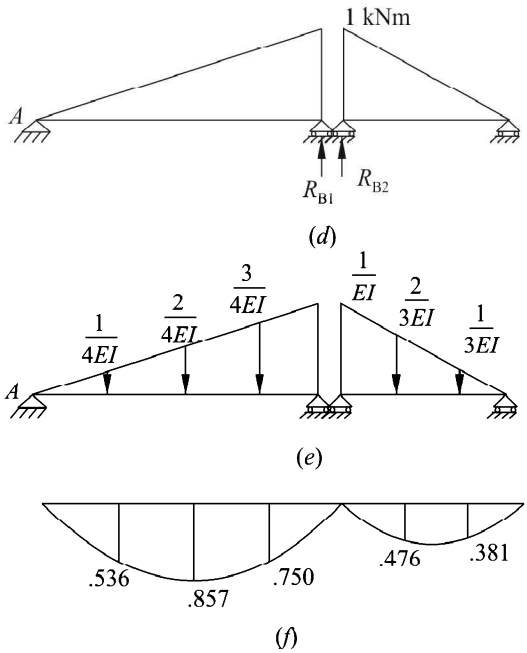


Figure 5.8(*d*): Bending moment diagram for released beam. **(e)** Conjugate beam. **(*f*)** ILD for moment at *B*.

Solution The restraint form moment at *B* is removed and unit moment is applied at B as shown in Figure 5.8(*b*). The released structure is as good as two independent simply supported beams (Refer Figure 5.8(*c*). The deflection of the released beam is to be found which is the influence line diagram for the continuous beam to some scale. The bending moment diagram for released beam is as shown in Figure 5.8(*d*). For finding deflection, conjugate beam method is used. The conjugate beam with $\left(\frac{M}{EI}\right)$ diagram load is as shown in Figure 5.8(*e*).

In conjugate beam,

$$\Sigma M_B = 0, \text{ gives}$$

$$R'_A \times 8 = \frac{1}{2} \times 8 \times \frac{1}{EI} \times \frac{8}{3}$$

$$R'_A = \frac{4}{3EI}$$

$$R'_{B1} = \frac{1}{2} \times 8 \times \frac{1}{EI} - \frac{4}{3EI} = \frac{8}{3EI}$$

$$R'_C \times 6 = \frac{1}{2} \times 6 \times \frac{1}{EI} \times 2$$

$$R'_C = \frac{1}{EI}$$

$$R'_{B2} = \frac{1}{2}\times 6\times\frac{1}{EI} - \frac{1}{EI} = \frac{2}{EI}$$

$$\delta_A = M_A = 0$$

$$\delta_1 = M_1 = \frac{4}{3EI}\times 2 - \frac{1}{2}\times\frac{1}{4EI}\times 2\times\frac{2}{3} = \frac{25}{EI}$$

$$\delta_2 = M_2 = \frac{4}{3EI}\times 4 - \frac{1}{2}\times 4\times\frac{1}{2EI}\times\frac{4}{3} = \frac{4}{EI}$$

$$\delta_3 = M_3 = \frac{4}{3EI}\times 6 - \frac{1}{2}\times 6\times\frac{3}{4EI}\times 2 = \frac{3.5}{EI}$$

$$\delta_B = M_B = 0$$

For portion BC, calculating bending moment from the right end, we get,

$$\delta_C = M_C = 0$$

$$\delta_5 = M_5 = \frac{1}{EI}\times 2 - \frac{1}{2}\times 2\times\frac{1}{3EI}\times\frac{2}{3} = \frac{1.778}{EI}$$

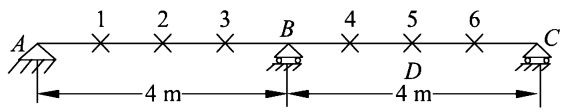
$$\delta_4 = M_4 = \frac{1}{EI}\times 4 - \frac{1}{2}\times 4\times\frac{2}{3EI}\times\frac{4}{3} = \frac{2.222}{EI}$$

Rotation α_B at B in a released structure

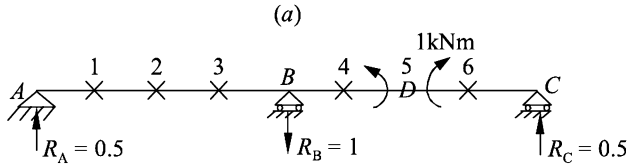
$$= \text{Reaction at } B \text{ in conjugate beam} = R_{B1} + R_{B2} = \frac{8}{3EI} + \frac{2}{EI} = \frac{4.667}{EI}$$

Ordinates of influence line diagrams are obtained by dividing the deflections by relative rotation $\alpha_B = \frac{4.667}{EI}$. Hence, *ILD* for M is as shown in Figure 5.8(*f*).

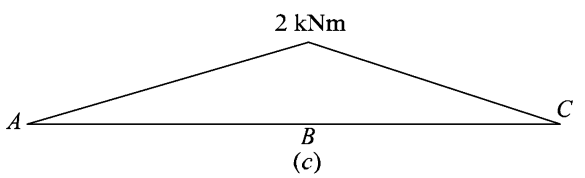
Example 5.4 Compute the ordinates of influence line for moment at mid-span of BC for the beam shown in Figure 5.9(*a*) at 1 m interval and drawn influence line diagram. Assume moment of inertia to be constant throughout.



(*a*)

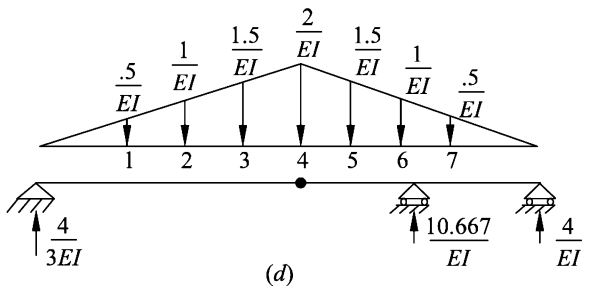


(*b*) Released beam with unit moment

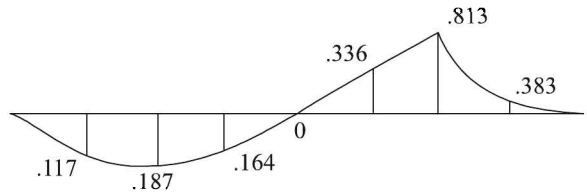


(*c*)

Figure 5.9(*a*): Given beam. **(*b*)** Released beam with unit moment at *D*. **(*c*)** Bending moment diagram for released beam.



(*d*)



(*e*)

Figure 5.9(*d*): Conjugate beam. **(e)** ILD for moment at *D*.

Solution Restraint for moment at mid-span of *BC* is removed by introducing hinges and unit moment is applied as shown in Figure 5.9(*b*). According to Müller-Breslau principle, the deflected shape of this released structure represents ILD for moment at that point.

In a released structure (Figure 5.9(*b*)),

$$R_C \times 2 = 1$$

$$\therefore \quad R_C = 0.5\,\text{kN}$$

Taking moment about *A*, we get,

$$R_B \times 4 - R_C \times 8 = 0$$

$$R_B = 2R_C = 2 \times 0.5 = 1\ \text{kN}\downarrow$$

$\Sigma V = 0$, gives

$$R_A = 1 - 0.5 = 0.5\ \text{kN}$$

Therefore, bending moment diagram for released beam is as shown in Figure 5.9(*c*).

The conjugate beam for the released beam and $\frac{M}{EI}$ loading on it are as shown in Figure 5.9(*d*).

In conjugate beam,

$\Sigma M_B = 0$, gives

$$R'_A \times 4 = \frac{1}{2} \times 4 \times \frac{2}{EI} \times \frac{4}{3}$$

$$R'_A = \frac{4}{3EI}$$

$\Sigma M_C = 0$, gives

$$R'_A \times 8 + R'_D \times 2 - \frac{1}{2} \times 8 \times \frac{2}{EI} \times 4 = 0$$

$$\frac{4}{3EI}\times 8 + R'_D \times 2 - \frac{1}{2}\times 8 \times \frac{2}{EI}\times 4 = 0$$

$$R'_D = \frac{10.667}{EI}$$

$\Sigma V = 0$, gives

$$R'_A + R'_B + R'_C = \frac{1}{2}\times 8 \times \frac{2}{EI} = \frac{8}{EI}$$

$$\therefore \quad R'_C = \left(\frac{8}{EI}\right) - \left(\frac{4}{3EI}\right) - \left(\frac{10.669}{EI}\right)$$

$$= -\frac{4}{EI} = \frac{4}{EI}\downarrow$$

Calculating moments from left end for portion AB, we get,

$$\delta_A = M_A = 0$$

$$\delta_1 = M_1 = \left(\frac{4}{3EI}\times 1\right) - \left(\frac{1}{2}\times 1 \times \frac{0.5}{4EI}\times \frac{1}{3}\right) = \frac{1.25}{EI}$$

$$\delta_2 = M_2 = \left(\frac{4}{3EI}\times 2\right) - \left(\frac{1}{2}\times 2 \times \frac{1}{EI}\times \frac{2}{3}\right) = \frac{2}{EI}$$

$$\delta_3 = M_3 = \left(\frac{4}{3EI}\times 3\right) - \left(\frac{1}{2}\times 3 \times \frac{1.5}{EI}\times 1\right) = \frac{1.75}{EI}$$

$$\delta_4 = M_B = 0$$

For portion BC, calculating moments in conjugate beam from end C, we get,

$$\delta_C = M_C = 0$$

$$\delta_5 = M_5 = -\left(\frac{4}{EI}\times 1\right) - \left(\frac{1}{2}\times 1 \times \frac{0.5}{EI}\times \frac{1}{3}\right) = -\frac{4.083}{EI}$$

$$\delta_D = M_D = -\left(\frac{4}{EI}\times 2\right) - \left(\frac{1}{2}\times 2 \times \frac{1}{EI}\times \frac{2}{3}\right) = -\frac{8.667}{EI}$$

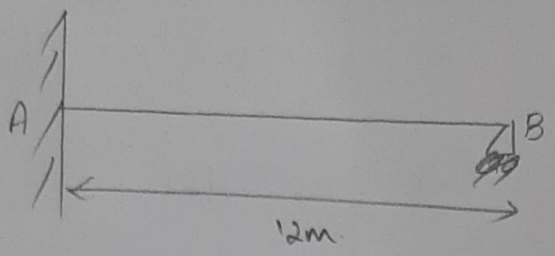
$$\delta_4 = M_4 = -\left(\frac{4}{EI}\times 3\right) - \left(\frac{1}{2}\times \frac{1.5}{EI}\times 3 \times 1\right) + \left(\frac{10.667}{EI}\times 1\right) = -\frac{3.583}{EI}$$

The total rotation at hinge D in a released structure

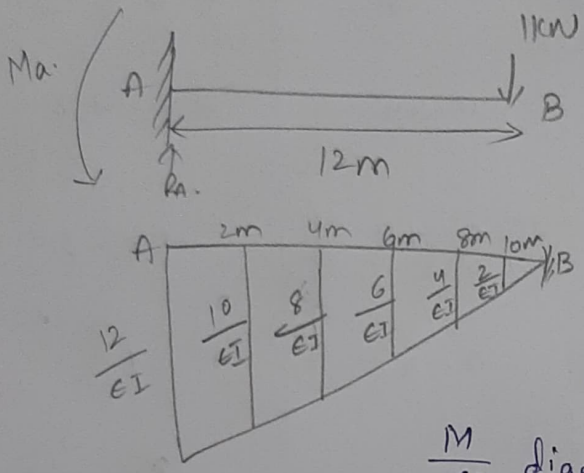
$$= \text{Reaction at support } D \text{ in conjugate beam} = \frac{10.667}{EI}$$

Hence, values of the influence line ordinates at each of the above sections is computed by dividing each deflection by $\alpha = \dfrac{10.667}{EI}$. Hence ILD for M_D is as shown in Figure 5.9(e).

7Q Draw the influence diagram for propped cantilever beam having Span AB of 12m



Sol:



From Muller - Breaslaw Theory.

let us Redundant $R_B = V_b$

$\Sigma V = 0 \Rightarrow R_A = 1$

$\Sigma M_a = 0 \Rightarrow M_a = 1 \times 12 = 12$

$\frac{M}{EI}$ diagram (Conjugate Beam)

In actual beam deflection value is equal to Bending moment in Conjugate Beam.

$\delta_A = \delta_{0m} = 0$

$\delta_{2m} = \frac{1^2}{6}(2a+b) = \frac{(2)^2}{6}\left(2 \times \frac{12}{EI} + \frac{10}{EI}\right) = \frac{22.67}{EI}$

$\delta_{4m} = \frac{(4)^2}{6}\left(2 \times \frac{12}{EI} + \frac{8}{EI}\right) = \frac{85.33}{EI}$

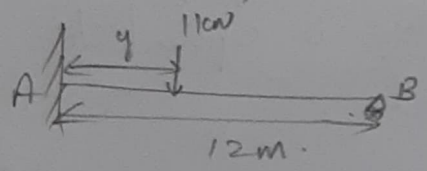
$\delta_{6m} = \frac{(6)^2}{6}\left(2 \times \frac{12}{EI} + \frac{6}{EI}\right) = \frac{180}{EI}$

$\delta_{8m} = \frac{(8)^2}{6}\left(2 \times \frac{12}{EI} + \frac{4}{EI}\right) = \frac{298.67}{EI}$

$\delta_{10m} = \frac{(10)^2}{6}\left(2 \times \frac{12}{EI} + \frac{2}{EI}\right) = \frac{433.33}{EI}$

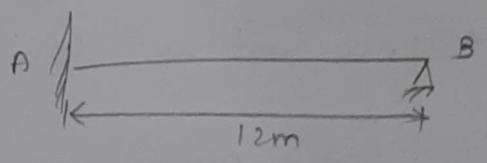
$\delta_{12m} = \frac{(12)^2}{6}\left(2 \times \frac{12}{EI} + 0\right) = 576/EI$

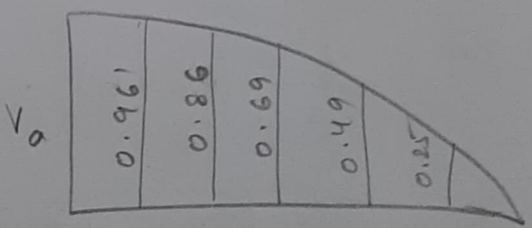
location from Support A	δ_x	Muller Breaslaw theorem $V_b = \delta_x / \delta_b$	$V_a + V_b = 1$ $\Rightarrow V_a = 1 - V_b$	$M_a = y - 12 V_b$
0m (A)	0	0	$1 - 0 = 1$	$M_a = 0 - 12 \times 0 = 0$
2m	$22.67/EI$	0.039	0.961	$M_2 = 2 - 12 \times 0.039 = 1.532$
4m	$85.33/EI$	0.14	0.86	2.32
6m	$190/EI$	0.31	0.69	2.28
8m	$\frac{298.67}{EI}$	0.51	0.49	1.88
10m.	$\frac{43333}{EI}$	0.75	0.25	1
12m (B)	$\frac{576}{EI}$	1	0	0

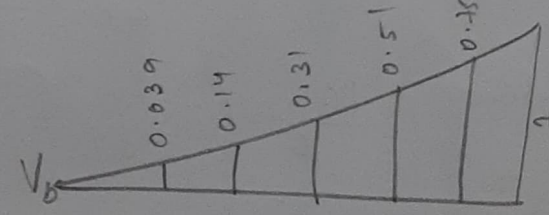


$M_a = 1 \cdot y - 12 \times V_b$

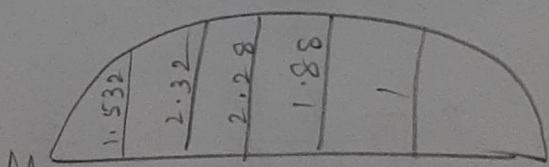
$M_a = y - 12 V_b$



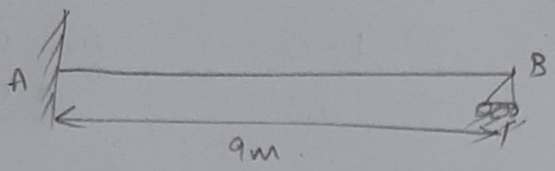


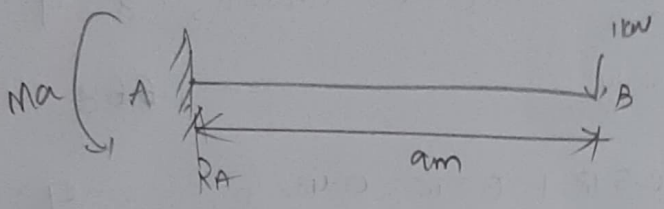


ILD Diagrams

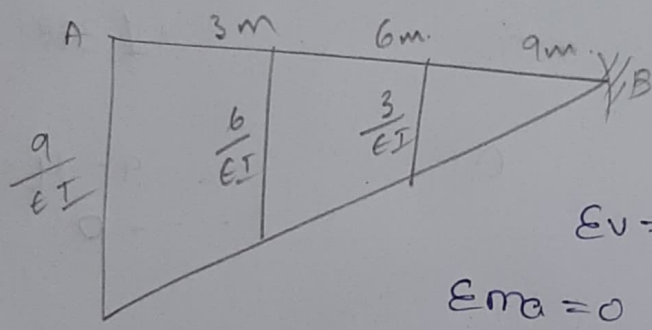


10 Draw the influence diagram for propped Cantilever beam having span AB of 9m.





From muller-Breslau theory



led us Redundant

$R_B = v_b$

$\Sigma v = 0 \Rightarrow R_A = 1$

$\Sigma m_a = 0 \Rightarrow m_a = 1 \times 9 = 9$

$\frac{m}{EI}$ diagram (conjugate Beam)

in actual beam value is equal Bending moment in conjugate Beam.

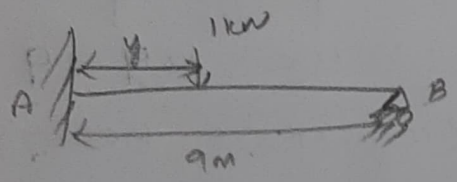
$\delta_A = \delta_{0m} = 0$

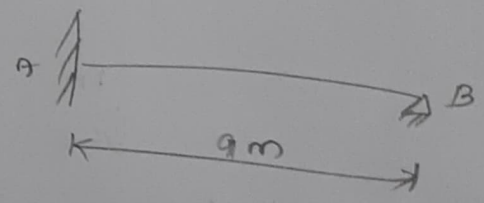
$$\delta_{3m} = \frac{d^2}{6}(2a+b) = \frac{(3)^2}{6}\left(2\left(\frac{9}{EI}\right) + \frac{6}{EI}\right) = \frac{36}{EI}$$

$$\delta_{6m} = \frac{(6)^2}{6}\left(2\times\left(\frac{9}{EI}\right) + \frac{3}{EI}\right) = \frac{126}{EI}$$

$$\delta_{9m} = \frac{(9)^2}{6}\left(2\times\left(\frac{9}{EI}\right) + 0\right] = \frac{243}{EI}$$

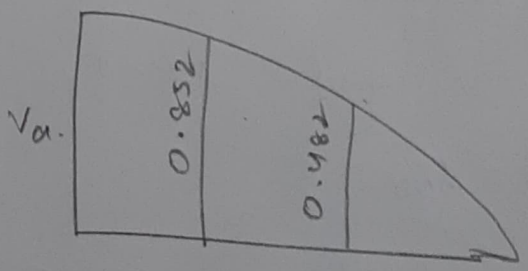
location from support (A) (y)	δx	Mullers Breeslaw theory $v_b = \delta x / \delta_b$	$v_a + v_b = 1$ $\Rightarrow v_a = 1 - v_b$	$m_a = y - q v_b$
0m	0	0	$1 - 0 = 1$	$m_a = 0 - q \times 0 = 0$
3m	$36/EI$	$\frac{36}{EI} = \frac{36}{243}$ $\frac{243}{EI} = 0.148$	$1 - 0.148 = 0.852$	$m_3 = 3 - q \times 0.148$ $= 1.668$
6m	$126/EI$	$\frac{126}{243} = 0.518$	$1 - 0.518 = 0.482$	$m_6 = 6 - q \times 0.518$ $= +1.338$
9m	$\frac{243}{EI}$	1	0	$m_q = 9 - q \times 1$ $= -q$

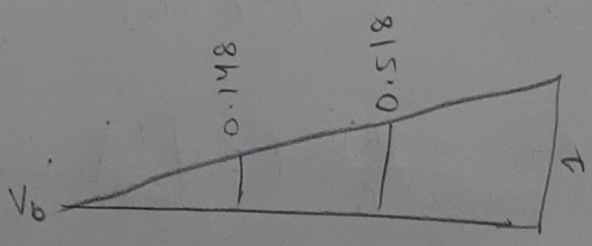


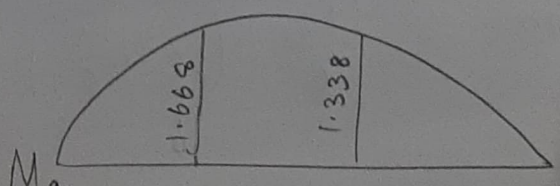


$m_a = 1 \times y - q \times v_b$

$m_a = y - q v_b$







I & D Diagrams